A COMPARATIVE ANALYSIS OF ITALIAN AND GREEK EUCLIDEAN GEOMETRY TEXTBOOKS: A CASE STUDY

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INTRODUCTION

School textbooks play an important role in mathematics education because of their close relation to the teaching and learning process (Valverde et al. 2002, Johansson 2003). Textbooks have a direct influence on both teachers and students. For the former, they are a source of curricular content, a paradigmatic exposition of an organised body of knowledge and a suggestion of didactical style. For the latter, they are an authoritative source of content and an important working tool to consolidate subject matter knowledge through their suggested activities. Textbooks reflect contemporary thoughts about mathematics education and reveal its aims, they provide a guide to what educators consider feasible or desirable and how goals might be attained, and in certain cases they attempt to establish new content or pedagogical norms (Howson 1995). In addition, textbooks express a specific perception of communication and pupil-teacher relationships and can also be a source of information for other interested parties, such as parents (Seguin 1989). Moreover, textbooks have moral and intellectual influence on their users (Johnsen 1993) who may develop through their contact with textbooks an attraction to books and the habit of using them as a means to enrich their knowledge (Seguin 1989).

Textbooks exert political and ideological influence even in disciplines such as mathematics and natural sciences, which are usually considered as politically and ideologically neutral (Apple 1992, Johnsen 1993). Ideology is an inevitable dimension of text content and has a decisive influence on the shaping of the forms of the text, as well as on the social and pedagogical interaction mediated by the text (Johnsen 1993). Textbooks may reflect official intentions and visions of national curricula, as well as the educational views and pedagogical values of the textbook designers and the decision makers of textbook selection (Schmidt *et al.* 1997). Since education was recognised as a universal right, the generalised use of textbooks has been considered indispensable in ensuring the efficiency of instruction and academic success. Especially in most of the developing countries, educational authorities have now realised that an increase in the production of good quality school textbooks is an important factor in improving instruction in schools (Seguin 1989).

The present study attempts a comparative content analysis of two Euclidean geometry textbooks; a leading Italian textbook used in many Italian 'lycei' and the unique official textbook currently used in the Greek 'lyceum'. Compared to other research areas in mathematics education, studies focusing on textbooks have been scarce (Howson 1995, Turnau 1983), and many researchers have urged for more relevant studies (Love & Pimm 1996). This study is undertaken in view of a renewed international interest in the geometry curriculum (Wu 1996, CREM 2000, The Royal Society/JMC 2001) with most countries looking for change (Hoyles *et al.* 2002, Fujita & Jones 2003).

The perceived aim of a comparative enquiry is a better understanding of the cultures involved, which provides a better perspective from which researchers can scrutinise tacit assumptions hidden in each culture, ensuring a deeper understanding of issues that are of central concern to each particular culture (Pepin & Haggarty 2001, Haggarty & Pepin 2001). Cross-national comparisons may also indicate possible hitherto unexplored directions that could be pursued, or 'they may help to sharpen the focus of analysis of the subject under study by suggesting new perspectives' (Pepin & Haggarty 2001, p. 158).

REVIEW OF LITERATURE

Van Dormolen (1986) distinguishes three types of textual analysis, *a priori* textual analysis, which consists in analysing a text as a possible means of instruction, *a posteriori* analysis concerned with comparing learning results with the text and *a tempo* analysis concerned with the way teachers and students use texts in the teaching and learning process. For the purposes of this study we shall be concerned with the former type.

A mathematical textbook can be analysed in terms of various aspects of its mathematical content. The TIMSS (Third International Mathematics and Science Study) curriculum and textbook analysis provided a systematic method of analysing and comparing the mathematics topics and other pedagogical characteristics collected from hundreds of textbooks of almost 50 countries (Foxman 1999). Part of the above analysis has sought to illustrate the similarities and differences between mathematics curricula in different countries as they are reflected in the textbooks used in the educational process. To achieve this goal a common international frame of reference was developed, which considered three aspects of textbooks: content, performance expectations and perspectives (Schmidt et al. 1997). The Eisenhower National Clearinghouse for Mathematics and Science Education (undated) describes these aspects as follows:

The *content* aspect represents the content of school mathematics partitioned into ten major categories: numbers, measurement, geometry (position, visualisation and shape), geometry (symmetry and congruence), proportionality, functions – relations –

equations, data – probability – statistics, elementary analysis, validation and structure, and other content. The *performance expectations* aspect describes the kinds of performance that students are expected to demonstrate while engaged with the particular content. Finally, the *perspectives* aspect is intended to depict curricular goals that focus on the development of students' attitudes, interests and motivations in mathematics teaching. This aspect makes it possible to describe curriculum materials that are intended to promote positive attitudes and mathematical modes of thought or habits of mind, as well as goals that are encouraging towards careers in mathematics, science or technology. Intended learning experiences that promote participation of groups currently under-represented are also included.

For the performance expectations aspect of the mathematics framework the following categories were used (Robitaille et al. 1993):

- *Knowing*: Representing, recognising equivalents, recalling mathematical objects and properties.
- *Using routine procedures*: using equipment; performing routine procedures, using more complex procedures.
- *Investigating and problem solving*: Formulating and clarifying problem situations, developing strategy, solving, predicting, verifying.
- *Mathematical reasoning*: Developing notation and vocabulary, developing algorithms, generalising, conjecturing, justifying and proving, axionatising.
- *Communicating*: Using vocabulary and notation, relating representations, describing/discussing, critiquing.

For the perspectives aspect of the mathematics framework the following categories were used (Schmidt et al. 1997):

- Attitudes towards science, mathematics and technology

- Careers involving science, mathematics and technology
- Participation in science and mathematics by underrepresented groups
- Science, mathematics and technology to increase interest
- Science and mathematical habits of mind.

To conduct an in-depth analysis of textbooks, a two-step process was used: (1) each document was divided into major structural components called *units*, which were subdivided into smaller segments within units, called *blocks* (2) each of these units and blocks was then assigned content, performance expectation, and perspective category codes (US Department of education 1997). Each framework aspect was organised hierarchically into a nested subcategory system of increasing specificity. This is system was flexible in the sense that more than one framework subcategories could be assigned to each particular text segment. The units and block types used were (Schmidt et al. 1997):

UNIT TYPES

- Lessons
- Multiple lessons
- Introductions
- Instructional appendices
- Other

BLOCK TYPES

- Narrative blocks (central, related, unrelated, and sidebar instructional narrative)
- Graphic blocks related to narrative
- Informative graphic blocks not directly related to instructional narrative
- Exercise and question sets

- Suggested activities
- Worked examples
- Other

Another approach to mathematical textual analysis was proposed by Van Dormolen (1986) who views mathematics as an activity arising from problem situations. By means of this activity, new knowledge is created and communicated in textual form consisting of definitions, theorems, rules, methods and conventions. The above elements of mathematical knowledge he calls *kernels*. Categorising the kernels contained in a text, Van Dormolen (1986) distinguishes the following textual aspects: a *theoretical aspect* consisting of theorems, definitions and axioms; an *algorithmic aspect* containing explicit 'how to do…' rules; a *logical aspect* consisting of statements informing the reader about how s/he is allowed to handle theory; a *methodological aspect* consisting of all heuristic hints; and, finally, a *communicative aspect* emphasising commonly adopted conventions.

Most of Van Dormolen's analytic categories stated above can be subsumed to the analytic categories developed in the TIMSS mathematics framework. On the other hand, the TIMSS analysis is concerned mostly with topic coverage, performance expectations and perspectives, and not with the pedagogical approaches that textbook authors adopt to facilitate the student's acquisition of the required knowledge and skills; Van Dormolen's methodological aspect touches on this issue. The methodological aspect is especially important as an aid to the problem solving activities contained in a textbook. These activities are essential to the learning process because they consolidate and fix the acquisition of knowledge and the mastery of concepts and are a means of evaluating student progress (Seguin 1989). Moreover, the range of exercises and their potential for promoting learning, understanding, retention, discovery and motivation are an important pedagogical textbook element (Howson 1995).

Gerard and Roegiers (2003), distinguishing the different kinds of textbook exercises in terms of their intended function, classified them into the following categories: application exercises, comprehension exercises, consolidation exercises, remediation exercises and advanced exercises. With regard to the various types of problems, Fan and Zhu (2004) provided a conceptual framework, according to which problems can be classified by means of the following categories: routine versus non-routine problems, traditional versus non-traditional problems, open-ended versus close-ended problems, application versus non-application problems, single step versus multiple step problems, sufficient data problems versus extraneous or insufficient data problems, and problems in pure mathematical form versus problems in verbal, visual or combined form.

Except for the above aspects of textbook mathematics content, Seguin (1989, p.25), in his methodological guide for the elaboration of textbooks, proposed an analysis of the nature of textbook content according to the following categories:

- *Accuracy*: content should be free from errors
- *Precision*: content should not be incomplete or ambiguous
- *Topicality*: information presented should be up-to-date and correspond to present day realities
- *Objectivity*: content should not convey distorted or incomplete knowledge or information due to an ideological bias or dogmatic standpoints
- *Interdisciplinarity*: integration of contents from more than one disciplines, permitting inter-connection of phenomena of a complex nature that cannot be satisfactorily explained from the point of view of a single discipline and

contributing to developing the child's capacity to discern relationships between phenomena which appear to be totally unconnected.

- *Progression of concepts and development of abilities*: progression of content and learning activities should lead the pupil towards knowing how to use simple concepts, and then more complex ones, in order to master their application to a variety of situations or facts.
- *Contribution to social objectives*: content should promote social and moral attitudes favourably disposed to community life and should contribute to the development of an appreciation of social, moral and aesthetic values

Seguin's last category may be considered irrelevant by those who view mathematics as a neutral and value free discipline. A growing number of educators, however, have suggested that mathematics is value-laden (Ernest 1998). Moreover, each of the different philosophies of mathematics has its own impact on educational practices (Ernest 1991). Consequently, each textbook reflects a particular view of mathematics. A textbook, by its content and structure and the kinds of activities it requires, gives a particular picture of what mathematics is as a creative activity (Howson 1995). Seen from different perspectives, mathematics may be regarded as a set of truths and rules, a study of patterns that can be discerned experimentally, a rigidly deductive discipline, a senseless formal game, the language of nature, etc.

Another essential element of a textbook is its pedagogical aspect. A textbook's method may correspond to a constrained, interventionist pedagogy, or, on the contrary, it be inspired by open learning methods, inciting the pupil towards some autonomy in learning (Seguin 1989). The pedagogical features of textbooks have been analysed in the literature in terms of the following themes:

Organisational component: According to Van Dormolen (1986) a text must provide *cursory preparation* to the student, ensuring that there is a logical progression of the subject matter presented in the text; and *conceptual preparation* in the sense that a text's kernels must be presented in such a way that cognitive structures previously acquired by the student should help the student to assimilate new concepts.

Adaptation of mathematical content to students' abilities: Seguin (1989) argues that pupils have different levels of ability and learning rates, varying with age, their socio-cultural milieu, and their psychological traits. Designing a textbook for an average pupil will not address these disparities and variations. Therefore, 'more abundant and varied contents should enable the majority of pupils to understand the facts studied, the principles or general ideas introduced in a lesson or in a teaching unit' (Seguin 1989, p.27).

Motivation: According to Seguin (1989), elaboration and choice of content should take account of the fact that the interests of pupils are varied and differ according to age, environment, and psycho-affective traits. Moreover, in the presentation of content, pupils should find situations which are familiar to them, or contextualised examples drawn from their own environment.

Rhetorical voice of a text: Crismore (1989) suggests that the reader's learning can be facilitated by the author providing a meta-discourse to the text, reflecting the latter's presence in the text and making explicit the author's goals, main points and attitudes. This rhetorical voice of the text helps readers organise, classify, interpret, evaluate and react to the material (Van de Kopple 1985).

Pedagogical deficiency of the deductive method: Deductivity, once considered as the prototype of mathematical exposition and for some the logic of mathematical discovery, has been criticised on pedagogical grounds (Freudenthal 1973, Lakatos

1976). According to Lakatos (1976), the deductive style is authoritarian in character and distorts the picture of how mathematics is actually created by imposing artificial definitions, presenting theorems 'loaded with heavy-going conditions' making no clues as to how one would arrive at them, and 'suppressing the primitive conjecture, the refutations and the criticism of proof' leading into an 'end result … exalted into sacred infallibility' (Lakatos 1976, p. 142). The above critique is especially relevant to the present study, which undertakes an analysis of two traditional synthetic Euclidean geometry textbooks. What a comparative analysis of this feature in textbooks of such a kind can reveal is to what extend the perceived shortcomings of the deductive method stated above can be alleviated or perhaps even obliterated by a rich meta-discursive presence, which will uncover the real path that the intellect has followed to arrive at this material, a path that is now hidden either for perceived aesthetic purposes or for the sake of precision and simplicity, in any case in order to satisfy the requirements and values set and cherished by the mathematical community.

METHODOLOGY

The textbooks chosen for this comparative analysis are the fourth edition of *Il Nuovo Pensiero Geometrico* by Cateni, Fortini and Bernardi (2002) and the first edition of *Euclidean Geometry* by Argyropoulos, Vlamos, Katsoulis, Markatis and Polyhronis (2004). The former is popular in Italy (Villani – co-editor of *Perspectives on the teaching of geometry for the 21st century*, Dordrecht: Kluwer Academic Publishers, personal communication) and is usually adopted by schools for the teaching of plane geometry for those pupils who attend the ninth and tenth grade of the Italian scientific lyceum. The latter textbook is the unique official Greek textbook currently used for plane geometry by all pupils attending the tenth and eleventh grade in all streams of the Greek lyceum. Although both textbooks include chapters on solid geometry, these chapters will not be included in this analysis inasmuch they do not constitute part of the implemented Greek geometry curriculum. To ensure comparability of topic areas, I chose to analyse the first six chapters of the Italian textbook and the first five chapters of the Greek textbook excluding from the latter the sections referring to the circle, which is treated later in the former textbook.

Adopting a modification of the approach elaborated by the TIMSS team (Schmidt et al. 1997), I segmented the sample content of the two textbooks into blocks, which have been categorised as follows:

BLOCK TYPES

- Theorems, corollaries (enunciation, proof)
- Definitions
- Axioms
- Worked examples
- Graphic blocks
- Exercises
- Narrative blocks (introductory remarks, linking passages, pointers, methodology tips, theoretical remarks and clarificatory comments, examples and counter-examples, perspectives)
- Other

The perspectives subcategory refers to the nature of mathematical thinking, the practical uses of mathematics, careers in fields that make use of mathematics, attitudes towards mathematics and its importance, participation in science and mathematics by underrepresented groups, interdisciplinarity, etc.

Theorems and corollaries are divided into two blocks: the enunciation of the theorem or corollary and its proof. The two parts have a different function, inasmuch as the enunciation of a theorem or corollary must be memorised to be used in mathematical problems and activities and it would be assigned a different performance expectation from the proof of the theorem, which serves as a paradigm of mathematical reasoning. Moreover, in certain cases the proof of a theorem is omitted. The same is true of corollaries, whose proof is more often left to the reader.

Each definition counts as one block, so a paragraph containing many definitions counts for an equal number of blocks. This was done to facilitate comparisons because definitions are not uniformly distributed in the relevant paragraphs The narrative blocks category is divided into six subcategories most of which more or less comprise the meta-discursive part of the text discussed in the review of literature, or to put it differently, they belong to the peripheral writing (Shuard & Rothery 1984). The perspectives category is divided into seven subcategories, which address several of the themes discussed in the literature, such as objectivity, interdisciplinarity, contribution to social objectives, etc.

In classical Euclidean synthetic geometry textbooks, following Euclid's tradition, the main pattern of presentation had been, before the introduction of exercises by Greenleaf in 1858 (Herbst 2002), a repetition of the cycle definition – theorem – proof. The number of definitions and theorems in each textbook indicates the range of topics covered; while the number of axioms is an indication of rigour. The worked examples, as well as the narrative blocks reflect the textbook's didactical features.

Apart from the characterisations mentioned above, which are related to the block types, more characterisations related to the student performance expectations aspect were assigned to each block, using the TIMSS team categorisation cited in the review of literature (Robitaille et al. 1993) [except for a slight modification of the subcategory 'recalling mathematical objects and properties' where I added the term 'facts' to include all propositions and axioms]. The details for the exact content of each subcategory are supplied in the Appendix A:

The historical references, dispersed in small units throughout the Greek textbook but concentrated into two comprehensive sections in its Italian counterpart, were analysed separately. Instead of using a theoretical framework based on blocks, a more qualitative overall analysis was given in terms of how and to what extend the following themes were treated by the authors of the two textbooks:

- *Alternative solutions*, involving references and presentation of alternative solutions to problems treated in the main text.
- *Lives of mathematicians*, referring to the personal lives of mathematicians and their cultural milieu in which their work was produced.
- *Mathematics content not discussed in the text*
- *Implicit or explicit beliefs about the nature of mathematics throughout history*
- *Mathematics in social context*

Finally, the whole content of the two textbooks was scrutinised in order to detect any mathematical errors, ambiguities and inexactitudes.

To check for reliability, two colleagues currently teaching geometry at the Greek lyceum applied the analytic theoretical framework used in this study to the analysis of two different chapters of the Greek textbook. The same task was performed by an Italian science teacher teaching at the Greek gymnasium, this time for a chapter of the Italian textbook. The results did not show a significant deviation from the ones presented in this study.

RESULTS

Table 1 shows the number and the proportion of the different types of non narrative blocks for the two textbooks. The total number of blocks found in the relevant chapters was 1051 for the Greek textbook, occupying 120 of its 256 plane geometry pages and 1248 for the Italian textbook, occupying 95 of its 430 plane geometry pages.

Table 1: Proportion of non-narrative block types

Textbook	Theorems				Definitions		Axioms		Worked Examples		Graphic Blocks		Exercises	
	Enunciation		Proof		N	%	N	%	N	%	N	%	N	%
	N	%	N	%										
Greek	104	9.9	73	6.9	141	13.4	11	1	15	1.4	179	17	260	24.7
Italian	136	10.9	84	6.7	162	13	11	0.9	13	1	162	13	330	26.4

The results displayed in table 1 show a remarkable similarity between the two textbooks regarding the textbook space devoted to non-narrative block types. The big number of theorems and definitions indicates that the books are very dense in new ideas and facts. In view of the high demands that a synthetic geometry textbook poses on the readers, however, there is a very small number of worked examples, 1.4% of the block types for the Greek textbook and 1% for the Italian, which would serve as a prototype for student independent work. On the contrary, though it should be expected of geometry textbooks, there is an abundance of graph blocks, as aids to mathematical reasoning, in both books. In view of the TIMSS findings of a comparative analysis of textbooks from thirty-five countries, according to which roughly 50% of the blocks in all textbooks examined from all the grades were exercise and question sets (Schmidt *et al.* 1997, the textbook space devoted to exercises may seem very low (around 25%), especially since, in the present study, each exercise counted as one block. One should take into consideration, however, that in synthetic geometry textbooks most exercises are theorems to prove and, therefore, they are of high cognitive demand. Moreover, the textbooks examined in the TIMSS study were addressed to students of different age groups (age 9, 13 and 18) than the ones we are studying here (14 and 15 year olds for Italy and 15 and 16 year olds for Greece).

Table 2 shows the number and the percentage of the different types of narrative blocks for the two textbooks.

Table 2: Proportion of narrative block types

Textbook	Introductory Remarks		Linking Passages		Pointers		Methodology Tips		Theoretical remarks		Examples		Other	
	N	%			N	%	N	%	N	%	N	%	N	%
Greek	16	1.5	4	0.4	198	18.9	4	0.4	19	1.8	16	1.5	11	1
Italian	17	1.4	3	0.2	186	14.9	10	0.8	100	8	27	2.2	6	0.6

I think that the introductory remarks and linking passage blocks are rather few, reflecting the traditional Euclidean deductive style, in which the definitions and the enunciations of theorems are not motivated but imposed upon the reader (Kline 1961, Freudenthal 1973). The high proportion of pointers, that is blocks which provide clues and suggestions for the solution of problems, is due to the fact that both authorial teams provide hints to most of the textbook activities, especially the Greek authors. Methodological guidance is crucial for successful problem-solving, so its rarity in the two textbooks is probably meant to be compensated by the problem hints mentioned above. An aspect where the Italian textbook is much more comprehensive is the authors' meta-discursive presence. In fact, as is shown in table 2, the percentage of blocks devoted to theoretical remarks and clarificatory comments in the Italian textbook is 8%, which is more than four times the corresponding number in the Greek textbook. Finally, the number of examples in the two textbooks is rather small, especially in the Greek textbook.

Table 3 shows the space devoted by each textbook to the various performance expectations categories as they were elaborated in the TIMMS mathematics framework.

Table 3: Variation of textbook use of performance expectations

Knowing							Using routine procedures					
	Representing		Recognising Equivalents.		Recalling Math facts & properties		Using Equipment		Performing Routine procedures		Using More complex procedures	
Text	No.	%	No.	%	No.	%	No.	%	No.	%	No.	%
Gr.	179	17	9	0.9	151	14.4	179	17	18	1.7	13	1.2
It.	162	13	7	0.6	188	15.1	162	13	63	5	96	7.7
Investigating and problem solving												
	Formulating & clarify problem situations		Developing strategy		Solving		Predicting		Verifying			
Text	No.	%	No.	%	No.	%	No.	%	No.	%		
Gr.	9	0.9	6	0.6	3	0.3	0	0	6	0.6		
It.	2	0.2	7	0.6	0	0	0	0	5	0.4		
Mathematical Reasoning												
	Developing Notation vocabulary		Developing Algorithms		Generalising		Conjecturing		Justifying & prove		Axiomatising	
Text	No.	%	No.	%	No.	%	No.	%	No.	%	No.	%
Gr.	0	0	0	0	0	0	8	0.8	310	29.5	1	0.1
It.	5	0.4	0	0	7	0.7	4	0.3	217	17.4	8	0.6
				Communicating								
	Use Vocabulary & Notation		Relate Representations		Describe Discuss		Critique					
Text	No.	%	No.	%	No.	%	No.	%				
Gr.	177	16.8	0	0	14	1.3	2	0.2				
It.	192	15.4	0	0	52	4.2	15	1.2				

The textbooks' equal proportion of blocks pertaining to the 'recalling mathematical facts and properties' subcategory suggests that the two textbooks have comparable thematic richness. However, there is an important difference in the range and the level of opportunities offered by the two textbooks for the consolidation of subject matter and the possibility of applying it to a variety of problematic situations. Namely, only 2.9 % of the Greek textbook's blocks are devoted to performing simple, as well as more complex routine procedures, in contrast with the Italian textbook which devotes almost 13%. On the other hand, the Greek textbook, by devoting almost one third of its blocks to the 'justifying and proving' subcategory, emphasises tasks that require a formal proof of some mathematical fact, in contrast with the Italian textbook which devotes 17% to formal proof activities. Table 3 suggests that both textbooks provide little opportunity for investigating and problem solving activities. However, this conclusion might be due to the author's interpretation of the attributes that define each category. For instance, a proof task might as well be considered a problem solving activity, even though in this study it was not judged to fulfil the criteria (as they were elaborated by the TIMSS mathematics framework) for such a characterisation. Finally, the high proportion of the Italian textbook's blocks devoted to describing, discussing and critiquing (5.4%) as compared with the Greek textbook's rather low proportion (1.5%) of these blocks suggests that communication, which is considered as an essential part of mathematics and mathematics education contributing to the clarification, refinement and amendment of ideas (NCTM 2000), receives much more attention by the Italian authors.

The historical references occupy 98 pages in the Italian textbook, which constitutes 13.7% of the book's content and 16 pages in the Greek textbook, which is 4.6% of its content. The commonalities as well as the differences in the approach taken by the two authorial teams are the following:

The thematic range of the historical references in the Greek textbook is narrow being confined to such topics as the theory of parallels, measurement, the insoluble problems of antiquity, etc, which clarify theoretical subtleties of a specific chapter. On the other hand, the historical references and the anthology of critical readings compiled by the Italian authors cover a rich variety of themes: lives and contributions of mathematicians (Thales, Pythagoras, Euclid, Eudoxos, Archimedes, Tartaglia, Leonardo da Vinci, Galilei, Descartes), alternative solutions to problems treated in the main text (some salient examples will be annotated later), further theoretical developments such as Pasch's axiom and Saccheri's effort to prove Euclid's fifth postulate, mathematical fallacies such as pseudo-proofs based on incorrect diagrams, themes that are drawn from everyday situations such as the famous problem of the mirror's reflection, discussions on mathematics such as the distinction between pure and applied mathematics, as well as on the nature of mathematics with an annotated reproduction of the famous dialogue between Socrates and a slave on the nature of knowledge found in the Platonic *Meno*, philosophical and epistemological considerations such as an analysis of Cartesian rationalism and Kant's theory of knowledge, etc.

The depth, as well as the quality of the historical references is more profound in the Italian textbook. The narrative has clear objectives, is self-sufficient and provides an abundant variety of explanations. The latter feature is not shared by its Greek counterpart. For example, the historical sketch on the genesis and development of geometry in the Greek book is telegraphic (two pages) and suffers from a serious weakness appearing throughout the historical references, namely the use of terms and notions that are not defined or explained: transcendental number, collinear points, lemma, algebraic number, complex number, methods of Galois Theory, curvilinear triangle, equivalence classes, etc.

Two annotated passages of the Italian narrative follow below. The first gives an alternative formula for the area of a disc developed by the Egyptians:

A nice result obtained by the Egyptians is deducible to problem 50 in the book of Ahmes, in which it is explained how to find the area of a circular field having a diameter of 9 *khet*, where *khet* is a unit of length and *setat* the corresponding unit of measure. The following is written: 'Subtract from a diameter its one ninth, obtaining 8 *khet*. Multiply 8 by itself, obtaining 64. The area is 64 *setat'*. Observe that denoting by 2r the measure of the diameter and with A the area of the circle, the previous calculations are expressed by the formula (1) $A = (2r - \frac{2r}{9})^2 = (\frac{16}{9}r)^2 = 3.160...r^2$. We know that A is equal to πr^2, so we deduce that these ancient mathematicians approximated that number that we call π today by 3.16 which differs from π = 3.14...only by about 0.60%. Presumably, the formula (1) was derived by dividing a square of side 2r in 9 small rectangles (fig.1) supposing that is almost equivalent to the octagon ABCDEFGH, since 9 small squares minus 4 half-squares are 7 small squares, we get $A \cong 7\left(\frac{2r}{3}\right)^2 = \frac{63}{81}(2r)^2 \cong \frac{64}{81}(2r)^2 = \left(\frac{8}{9}2r\right)^2 = \left(2r - \frac{2r}{9}\right)^2$.

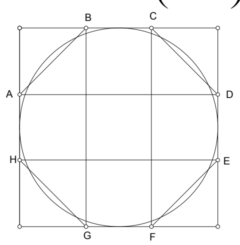


Figure 1

Figure 1: The Egyptian formula for the area of a disc

The text shown on figure 1 can be a source of a range of questions for classroom discussion:

- Do we need symbols to solve a problem?
- Does algebraic formalism offer any advantages?
- Is the Egyptian approximation adequate?
- Practically, does not even the standard formula A = πr^2 always give an approximation (of course with any degree of desired accuracy)?

- Maybe the Egyptians could have given a better approximation but they *chose* to be satisfied with their formula, as much as we are satisfied with our formula when we take the value of π equal to 3.14?
- Do other methods exist which give a good approximation of the area of the disc?
- Can the students devise such methods?

The second passage (Fig. 2) gives an experimental way to determine the value of π:

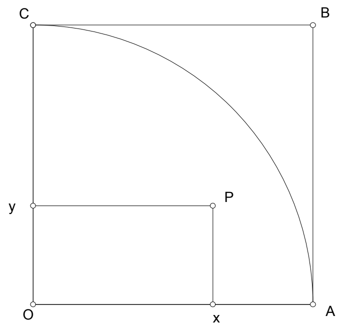


Figure 4

Let us look now at a method of determining the approximate value of π. The process we are proposing requires that we have a computer at our disposal, which is capable of generating random sequences; that is, to furnish a succession of numbers chosen at random from the interval [0, 1]. We draw a square Q = OABC of unit side (fig. 4) and the quarter circle S = OAC with centre O, whose area is π/4. We fix a Cartesian coordinate system as it is shown on fig. 4. Let P(x,y) be a point in the square Q, that is a point such that $0 \le x \le 1$ and $0 \le y \le 1$. The point P also belongs to S if and only if $(\mathrm{OP})^2 \le 1$ that is if and only if $x^2 + y^2 \le 1$. Choosing at random 100 pairs of numbers x, y, we know each time if the point belongs to S or not. If it belongs, let's say f times, the frequency f/100 furnishes an approximate value of the probability that a point in Q is also in S. On the other hand, this probability is equal to the ratio of the area π/4 of S over 1 of Q. We thus conclude that $\frac{f}{100} = \frac{\pi/4}{1}; \pi = \frac{4f}{100} = \frac{f}{25}$. By increasing the number of points P taken (something which is no problem with a computer) we obviously get better and better approximations of π.

Figure 2: An experimental way of finding π

The importance of this passage is that it tackles one classical Euclidean theme by following an approach which incorporates elements from different mathematical areas, namely, analytic geometry, statistics, probability theory and computer science.

It could even prompt students to device and participate in a game, an analogue of playing darts, where possibly blind-folded students would throw arrows targeting a figure similar to the one above. After the experiment (either way) students could discuss if the above approach constitutes an acceptable proof.

<u>Accuracy-Precision-Consistency-Clearness</u>

The Greek textbook contains a number of mathematical errors, inexactitudes, omissions and inconsistencies which are cited in the Appendix B. Of these flaws we reproduce here two examples:

In figure 3, the intuitive argument in part (c) that 'when M moves away indefinitely, the two line segments MA and MB tend to coincide, so the ratio MA/MB tends to 1', is actually wrong. In fact, the difference MA-MB instead of becoming arbitrarily small is constant and equal to AB. Here, the authors probably argued intuitively that since M moves away indefinitely, the two segments MA and MB are extremely large and because they differ by a fixed line segment, they are practically the same. It would be interesting to investigate if students who study this Greek textbook would identify this intuitive error.

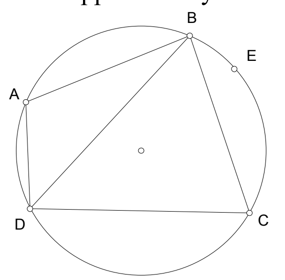
Limiting cases

(a) When M tends to A, MA tends to the null line segment, so MA/MB tends to zero.
(b) When M tends to B, MB tends to the null line segment, so MA/MB tends to infinity.
(c) When M moves away indefinitely, the two line segments MA and MB tend to coincide, so the ratio MA/MB tends to 1.

Figure 3: Incorrect intuitive reasoning

Figure 4 provides an example of erroneous reasoning in the proof of a theorem. In fact, the assertion ‘C is a point of the arc BED’ is a non-legitimate deductive step.

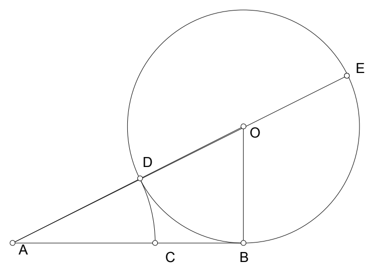
A quadrilateral is inscribable in a circle if two of its opposite angles are supplementary.



Let $\hat{A}+\hat{C}=2$ right angles. We draw the circle passing through A,B,D and the cord BD. The points A, C are on opposite sides of BD, so every inscribed angle subtending the arc BAD is equal to $\hat{C}$, whereas every inscribed angle subtending the arc BED is equal to its supplementary, that is $\hat{A}$. Hence, C is a point of the arc BED, so it belongs to the same circle as A, B, D.

Figure 4: An incorrect proof of a theorem

Before concluding the analysis of the two textbooks, one more issue has to be touched upon, which is related to the style adopted by authors writing synthetic geometry textbooks. Let us first read a theorem from the Italian textbook which best illustrates the case. It refers to the construction of the golden section:



Let AB be the given segment. From B we draw a line segment perpendicular to AB and equal to its half. Then we draw a circle with centre O and radius OB. We draw the line AO and let D, E be its points of intersection with the circumference. Finally, we transport the segment to AC. Then, the point C divides the segment AB in the required parts. In fact, since the tangent is the middle proportional between the entire secant and its exterior part (n.4), we have: AE:AB = AB:AD. Applying the rule (n.5, chap. XIII), we find (AE-AB):AB = (AB-AD). By construction, we have AB=2OB=DE, so we get AE-AB=AE-DE=AD=AC. Also, AB-AD=AB-AC=CB. Substituting, we have AC:AB=CB:AC and by the property of inverting (n.5, chap. XIII), we get AB:AC=AC:AD

Figure 5: The geometric construction of the golden section

Here, to quote Love & Pimm (1996 p.372), ‘the author is self-cast as the authoritative/authoritarian giver of instruction(s). “do this”, “do that; do it in this order, in this way” (“and”, we hear, *sotto voce*, “all will be well”)’. In this case, the Greek textbook partly provides an illuminating rationale for the original construction of the circle with radius AB/2. By letting $AB = a$ and $AC = x$, we have $\frac{AB}{AC} = \frac{AC}{CB} \Leftrightarrow AC^2 = AB \cdot CB \Leftrightarrow x^2 = a(a-x) \Leftrightarrow a^2 = x(x+a)$. From this point onwards, it is left to the reader to observe that the last relation is structurally the same as the formula relating the tangent segment and the secant segments in a circle with diameter a and hence of radius a/2. The issue here is whether authors should expect the reader to demystify a proof on his/her own, or whether this task would require the mediation of the instructor. I think that a third alternative would be the full elaboration of a restricted number of selected theorems in a way that defies logical order but identifies the central ideas and the basic lines of attack, so that the reader is taught how to unravel the proofs that are written in the conventional fashion.

CONCLUSION

The comparative analysis of two synthetic geometry textbooks from different countries but with similar mathematical education cultures has revealed certain similarities that the two books share, as well as some differences. The traditional Euclidean deductive style with its linear logical structure is respected by both authorial teams. However, while the pedagogical intervention is rather minimal in the Greek textbook, at least comparatively, the larger textbook space devoted by the Italian textbook to methodological guidance and commentary on theoretical points, as well as to clarificatory remarks, shows evidence of the Italian authors’ stronger meta-discursive presence. Moreover, the historical references of the Italian textbook, designed to put the mathematical achievements into a historical and cultural perspective, are more extensive and have more depth and better quality than those of its Greek counterpart. Both books, however, lack a richer variety of suggested activities, such as (1) contextualised examples ranging from simple applications of theory to investigation and problem solving activities (2) tasks that would enhance the ability to explore and make conjectures especially in dynamic geometry contexts (Hoyles & Jones 1998) and (3) activities integrating more than one disciplines especially in view of the fact that geometry has pervaded art as well as most modern fields of scientific research.

To my knowledge, the Italian textbook has no mathematical errors. On the contrary a series defect of the Greek textbook is that it contains several mathematical mistakes, omissions and inconsistencies, especially in the beginning chapters where the axiomatic part is explicitly developed.

This comparative analysis had to face certain incongruities that should be taken into consideration, the first having to do with the fact that the textbooks are addressed to students of a slightly different age, and the second owing to the fact that the Italian textbook is addressed to students who attend the scientific lyceum, whereas the Greek textbook is addressed to all the pupils attending all types of lyceums.

The present study has left out some aspects of *a priori* content analysis. An improved version of the study would include a sociological analysis of the textbooks content, which would provide useful information on the extent to which stereotypical attitudes and cultural norms may have influenced the textbooks authors (Dowling 1996). Moreover, a linguistic analysis concerning mainly vocabulary, phrases, their construction, their length, punctuation, and style of writing, referred to in the literature as the *readability* of a text (Shuard & Rothery 1984), could be used as a tool for evaluating the possible level of linguistic comprehension of the textbooks by the students. Also, in view of the importance of the tasks designed for student practice, an improved version of this study would include a cognitive analysis and subsequent categorization according to the level of difficulty of the textbooks' numerous cognitively demanding suggested problems and activities.

Finally, since this was an *a priori* textual analysis, which analysed the texts as possible means of instruction, we were not concerned about the relationship of textual content with the intended and the implemented curricula of the two countries, or about the way the textbooks are used by the instructors or the students, or with how student evaluation would influence the amount of emphasis given by the students on the various parts and content aspects of the two textbooks. These important issues need to be elucidated by further research.

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APPENDIX A

DETAILED MATHEMATICS FRAMEWORK CATEGORIES

FROM

ROBITAILLE ET AL. (1993) CURRICULUM FRAMEWORKS FOR

MATHEMATICS AND SCIENCE

- *Knowing*:

Representing (demonstrating knowledge of a nonverbal mathematical representation of a mathematical object or procedure either by selection or by construction, either formal or informal; representations might be concrete, pictorial, graphical, algebraic, etc.)

Recognising equivalents (selecting or constructing mathematically equivalent objects [e.g., equivalent common and decimal fractions; equivalent trigonometric functions and power series; equivalent representation of concepts – e.g., place value; equivalent axiomatic systems; etc.])

Recalling mathematical objects and properties (fitting given conditions)

- *Using routine procedures*

Using equipment (using instruments, using calculators and computers)

Performing routine procedures (counting and routine computations, graphing; transforming one mathematical object into another by some formal process, e.g., multiplying by a matrix; measuring)

Using more complex procedures (estimating to arrive at an approximate answer to a question; collecting, organising, displaying, or otherwise using quantitative data; comparing and contrasting two mathematical objects, quantities, representations, etc.; classifying objects or working with the properties with the properties underlying a classification system)

- *Investigating and problem solving*

Formulating and clarifying problem situations (formulate or clarify a problem related to a real world or other concrete situation)

Developing strategies (develop a problem-solving strategy or data-gathering experiment and discuss that strategy or experiment [not just applying the strategy or carrying out the experiment])

Solving (execute some known or ad hoc solution strategy)

Predicting (specify an outcome [number, pattern, etc.] that will result from some operation or experiment before it is actually performed)

Verifying (determine the correctness of the result of problem solving; interpret results in terms of an initial problem situation to evaluate how sensible the results are, etc.)

- *Mathematical reasoning*

Developing notation and vocabulary (develop new notation and vocabulary to record the actions and results of dealing with real-world and other problem situations)

Developing algorithms (develop a formal algorithmic procedure by performing a computation or solving a problem of a certain type)

Generalising (extend the domain to which the result of mathematical thinking and problem solving is applicable by restating results in more general and more widely applicable terms)

Conjecturing (make appropriate conjectures and conclusions while investigating patterns, discussing ideas, working with an axiomatic system, etc.)

Justifying and proving (provide evidence for the validity of an action or the truth of a statement by an appeal to mathematical results and properties or by an appeal to logic)

Axiomatising (explore a formal axiomatic system by relating subsystems, properties, or proposition in the system; consider new axioms and their consequences; examine the consistency of axiom systems, etc.)

- *Communicating*

Using vocabulary and notation (demonstrate the correct use of specialised mathematical terminology and notation)

Relating representations (work with relationships and related mathematical representations to show the linkages between related mathematical ideas or related mathematical objects)

Describing/discussing (discuss a mathematical object, concept, pattern, relationship, algorithm, result, or display from a calculator or computer)

Critiquing (discuss and critically evaluate a mathematical idea, conjecture, problem solution, method of problem solving, proof, etc.)

APPENDIX B

FLAWS FOUND IN THE GREEK TEXTBOOK

Ambiguous, vague and imprecise statements: (the problematic point is underlined)

‘By means of the perimeter of regular polygons, in what follows <u>we are approaching the concept</u> of the length of a circle’.

‘A segment with which we <u>compare</u> all segments is called the unit segment’.

‘On a line we take the <u>consecutive points</u> A, B, C.’

‘The angle of two secants is a function of the <u>inscribed angles formed by its sides and the circle</u>’.

Theoretical inadequacies:

1. The authors follow a naïve approach [in Halmos’ (1974) sense] when using equalities involving line segments. Thus, they subtract equalities; multiply both sides of an equality by a number, etc, without having stated the relevant properties regarding the equality relation. This could have been remedied by a note stating that, in this particular situation, the same properties hold with line segments as with real numbers.
2. The sum of two angles is not well defined. In fact, because an angle exceeding 360^0 is not defined, two non-convex angles cannot be added.
3. To prove the assertion that if the distance of the centre of a circle form a line is less than the radius of the circle, then the circle and the line have two points in common, the following proposition is used which itself needs justification: If A is an interior point of a circle and B is an exterior point, then the half-line AB has a

unique common point with the circle. In the Italian book, the latter assertion is postulated.

4. In certain theoretical developments some special cases are not considered. For example, the product nAB of a natural number n times a line segment AB is defined and then the equality $EZ = nAB$ is written as $AB = \frac{EZ}{n}$, which makes no sense for $n = 0$. Also, when proving the similarity of two triangles which have two equal angles respectively, the case where the two triangles are equal is not considered.
5. In the following definition except for an apparent misprint, the expression 'outside its extremes' is unclear: 'The points of a segment that lie *outside its extremes* are called interior points of the segment.' In another definition, 'quantity' is defined to be 'anything that is susceptible to increase or decrease'. According to this definition, 'anger' or 'pain' are quantities.
6. In an activity it is asked to prove the commutative and associative property of addition of line segments. But such properties can only be postulated.