# Mathematical problems with data option

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**Abstract**

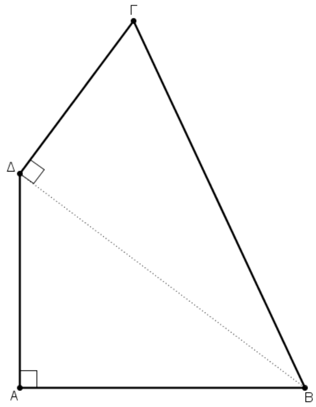
In many daily problems, that their treatment does not necessarily require mathematical knowledge, while we have a set of data, we use only a part of them. Which we use depends on how you plan to tackle the problem. Different workarounds require different combination of data.
Transferring and embodying the element of data selection, as this resulting from the real problems, we create a new kind of mathematical problems. In a problem of this nature, some of the data needed to solve it, selected through a data set, by the same student. The optional data are such that different combinations of options, allowing different strategies resolving and even each strategy resolving marks and different level mathematical thinking.

## 1. Introduction

«Nick wants to build the kitchen cabinets in his house. The measurement of the kitchen's dimensions of the specialist costs 50€. He has decided on the kind of wood and after market research knows that wood costs 800€ and the cutting of the pieces needed 200 €. The mechanical components (hinges, knobs) cost 100 €. If he buys the cabinets assembled will pay 1800€, if places them a specialist 300€.
Nick estimated he was able to measure the dimensions of the kitchen and assemble cabinets, but he could not cut the timber, nor to place the cabinets. So he decided to buy the timber, to give it for cutting, to buy the mechanical components and to call the specialist for placement. Cost 800+200+300+ 100=1400€».

«Dmitris student of second class of middle school, tomorrow has math and is trying to solve the following problem:

*We enclose the garden ΑΒΓΔ in the figure below, with wire mesh which is sold in rolls of 3 meters, each costs 25€. Given AB=12m, BΓ=17m and BΔ=15m. To calculate the cost of fencing.*




Dimitris thinks that if he calculates the sides AΔ and ΓΔ he can then solve it. He focuses on the data for a while and then says: I will apply Pythagorean theorem in the ABΔ to calculate the AΔ and after, Pythagorean theorem in the BΓΔ to calculate ΓΔ».

Looking at these two fantastic stories, we would say that a similarity is that in both there is a problem and a person involved trying to solve it. A difference is that in the first story Nick tries to solve a real problem, while in the second, Dimitris tries to solve a mathematical problem.
Nevertheless, we discern a difference in the structure of the two problems, which we consider important because it determines the behavior of the person involved as to the workaround. In the first story, Nick had a series of data available, he estimated which would take into account and based on it he solved the problem. Conversely, in the second story Dimitris did not have the selection of data, so he had to use all the data he had at his disposal to solve the problem.

In this paper, we propose a new kind of mathematical problems. We call *mathematical problems with data option* precisely because their nature is determined by the element of data selection, as deriving from the real problems.

## 2. The element of data selection from the daily problems in mathematical problems

There is a feedback between real life and teaching of mathematics. On one hand, the daily situations feed with ideas how we teach mathematics , on the other, what mathematics is taught the students has an impact on creating sustainable ways to adapt in areas of their experience and addressing daily problems, not only during their school life but also later on.
In many problems of daily life, while we have a set of data, we use only a part of them. Which one we use, depends on how we plan to tackle the problem. Different workarounds require a different combination of data. Among the workarounds, the available data and the mechanism of thinking where develops the involved, discern a relationship of interdependence and interaction.
The idea is to transfer and to embody in the design of mathematical activities the element of data selection. In the problems we suggest, some of the data required to solve them are selected through a data set, by the same student. The optional data is such that different combinations of options, allowing different strategies resolving and indeed, each strategy resolving marks and different level of mathematical thinking.
The element that distinguishes the proposed problems is the non-imperative use of all the data. Therefore is abolished the relationship correspondence between the data set and the solution and ceases to be valid the one-way that connects them and causes automated students' actions. So this gives space for selection and the selection process presupposes interpretation and decision making.

According to Glasersfeld (1983) an organism that simply acts and reacts without reflection, it will be wrong to say that interprets. The interpretation intimates understanding of more than one possibility, circumspection, feasibility and rationally controlled selection. In addition, to the student's ability to solve mathematical problems, we are interested and able to monitor their own actions. It's not enough to know what one does, but to know why what one is doing is right. This is why the teaching of mathematics is important to target in the conscious student understanding of what they do and why they do it. If anything characterizes the cognitive organisms is the ability to learn. In Piaget's terms the mathematical knowledge is "functional" rather than "digital", that is not an associative retrieval of a specific answer, but rather knowledge of what to do to produce an answer. The functional knowledge is constructional and is therefore best described in perceptual situations, deliberate choice and decision making.

The nobelist in medicine Edelman (1992), argues that the process of selection is typical of the brain and this ability is that differentiates persons towards learning. He suggests a selective theory of brain function, theory neuronal groups selection (TNGS) which explains the continuous adaptive matching organisms in the events of the environment, even these events cannot be predicted, that is, even when constituting novelties of environment.

All the above reinforce our intention to create mathematical problems that their solution requires selecting data. The embodiment of the element of data selection, aims to create situations control and coordination of knowledge, deliberate choice and decision making. The aim is the solution to be produced through conscious understanding of what I do and why I do it, despite to be schematized through hauling and matching stored knowledge For the teaching which has as a key component the proposed problems, primary importance is the epistemological principle of adaptation, namely of selection, despite of matching.

## 3. The example daily problem as a guide

The problem of constructing cabinets, will act as a guide for the transition to mathematical problems with data option. Restate the problem counting the data and look at how the selection of data has an impact to our approach to four different people.

***Real problem***

*The problematic situation involves the construction of cabinets in the kitchen of our house. We have already decided on the kind of wood and after market research we have the following data:*

1. *Measuring kitchen's dimensions by specialist: 50 €*
2. *Buy assembled cabinet: 1800 €*
3. *Installing cabinets by specialist: 300 €*
4. *Timber market: 800 €*
5. *Cutting wood pieces that need: 200 €*
6. *Mechanical components (hinges, knobs): 100 €*

We describe the workaround four different people:

- The A estimates that he can measure the dimensions of the kitchen, but he does not have the capacity neither to cut the timber, nor to assemble cabinets, nor to place them, so he decided to buy ready-made the cabinets and pay the specialist to place, cost 1800+300=2100€.
- The B believes that he can measure the dimensions of the kitchen and assemble the cabinets, but cannot even cut the wood, nor to place the cabinets, so he decides to buy the wood to give it to cut, to buy mechanical components and to call the specialist for placement, cost 800+200 +300 +100 =1400€.
- The C believes that he can measure the dimensions, assemble and place the cabinets, but cannot cut the timber. He decides to buy mechanical components and the timber and to give it for cutting, cost 800+200+100 =1100€.
- The D believes he can do it all by himself. He decides to buy only the timber and mechanical components cost 800+100 =900€.

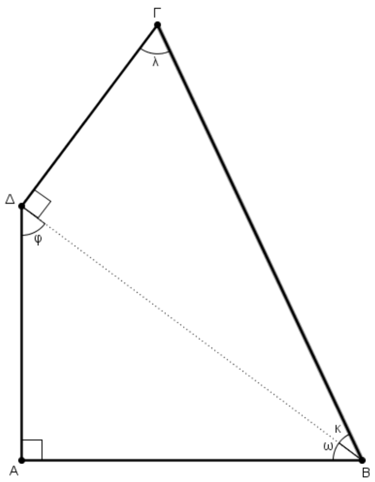
We distinguish four different solving strategies with different cost the each and we observe that any strategy uses a different combination of data. The cost for each data we can imagine as access cost, while the cost of each strategy resolving as the sum of the cost of accessing data using. Judging the ability of each involved to the construction of cabinets through the way he faced the problem, we observe that the cost of the strategy implemented marks the level of ability. The lower the cost the higher the ability level.

## 4. The transition to the mathematical problems with data option

Embodying the element of data selection and the contract of access cost, as they result from the previous real problem, we reconstruct the mathematical problem of introduction and convert it to a problem with data option as follows:

***Mathematical problem with data option***

*We enclose the garden ΑΒΓΔ in the figure below, with wire mesh which is sold in rolls of 3 meters, each costs 25 €. To calculate the cost of fencing.*




*You can find out:*
- *The length of the side AB=.......... (cost 3)*
- *The length of the side AΔ=.......... (cost 3)*
- *The length of the side BΓ=........... (cost 2)*
- *The length of the side ΓΔ=........... (cost 2)*
- *The measure of the angle* $\kappa$*=........... (cost 3)*
- *The measure of the angle* $\lambda$*=........... (cost 3)*
- *The measure of the angle* $\omega$*=.......... (cost 2)*
- *The measure of the angle* $\varphi$*=.......... (cost 2)*

*To resolve the problem at the lowest possible cost.*

To solve the problem must to use definitely the initial data, but we need also some by the optional data. To gain access to any of these, it must "shoulder" the corresponding "access cost". The initial data along with the selected data are the data requirements of a strategy resolving. The sum of the access costs is the "cost of resolving" of this strategy. Aim for the students is to solve the problem at the lowest possible cost.

This problem is addressed to students of second class of middle school. The learning target is the students to can combine properly the Pythagorean theorem and the trigonometric numbers.

The lowest cost of solution is 6 and corresponds to two strategies: seeking access to 3 data, the length of the sides BΓ and ΓΔ and the measure of angle ω or φ, apply the Pythagorean theorem in the triangle BΓΔ for to calculate the side BΔ and then appropriate trigonometric numbers in the triangle ABΔ for to calculate the sides AB, AΔ.

Overall, the different ways resolving are 41. If we categorize them by based on what basic mathematical knowledge or what combination of knowledge require them application, resulting the following four categories:

1) Request access to all four sides of the garden and not to use or Pythagorean theorem or trigonometric numbers, one way resolving with cost 10.
2) Request access to three sides and only use the Pythagorean theorem, 4 ways resolving with minimal cost 7 and maximum 8.
3) Request access to a side and two angles (one of a triangle and one of the other) and only use trigonometric numbers, 16 ways resolving with minimal cost 7 and maximum 8.
4) Request access on two sides and an angle and use one combination Pythagorean theorem and trigonometric numbers, 20 ways resolving with minimum cost 6 and maximum 9.

To achieve the minimum cost the student should not only make use of Pythagorean theorem and trigonometric numbers, but also combine them appropriately.

In the proposed problems, the element that creates the multiple strategies resolving, exactly is the possibility of data selection. The multiple ways resolving arise because we do not need the set of the data to solve the problem.

## 5. The contract of the access cost to the data

The contract of access cost to the proposed problems seems to modulate as motivation the reduction the cost of resolving. The strategies resolving are prioritized through a mechanistic criterion, the cost. The question is whether the contract of the access cost creates conditions so that the mechanism learning to take place in strictly behavioral contexts and therefore be incompatible with the constructivist approach to learning. The satisfaction of students through the learning process for finding the optimal strategy is pumped only through the reduction of cost resolving or also of the organization and adjustment achieved with their own construction? In fact perceive a strategy as optimal only because of the minimum cost and not because of higher level mathematical thinking required its application?

If we define as behaviorism the kind of associationism who considers that learning occurs through the connection muscular reactions to external stimuli (Βοσνιάδου 2001), we believe that the contract of access cost does not lead the learning in behavioral context and we consider, for the proposed problems, initially at least the access cost is necessary. Our view is reinforced by the interpretation of Glasersfeld for the aid and the behaviorism and the theory of didactical situations of Brousseau.

The Glasersfeld (1983) states that the basic tenet of behaviorism simply says their behavior is determined by the consequences it had produced in the past, namely that organisms operate inductively and I agree on this. The problem arises from the usual interpretation of "aid". There is a widespread misconception that aid is the result of well-known rewards, such as money or social acceptance. It is a misconception, not because their bodies will not work hard enough to get that money, but because it conceals far more reinforcing for a cognitive organization: to achieve a satisfactory organization, a sustainable response to a field of experience. This adds a different dimension to the concept of aid because any consequence of reward produced entirely within the body system. The self - creation, as aid has tremendous potential for cognitive organisms. We have spent valuable time and sweat of riddles that their solution did not yield neither money nor social acceptance.

The Brousseau (1986), in the context of the theory of teaching situations, notes that:

> "The knowledge are not there and have no meaning for the student, but only because they represent the best possible solution to a system of contracts. .... The teaching tasks is to organize these contracts and maintain the best possible conditions interactions " (pp. 368-369).

The Sierpinska and Lerman (1997) report that at the basis of the Brousseau's theory, is the epistemological assumption that knowledge exists and makes sense for the subject that is in cognitive process, only because it represents an optimal solution in a system of constraints. For a given target concept, the task of the teacher is to organize situations or systems of constraints for which this given concept will appear as an optimal (less costly) solution.

## 6. Thoughts, perspectives and questions

We mention some thoughts, perspectives and questions arising from the nature of mathematical problems with data option.

- An advantage of the proposed problems is, that allow students to involve with the same problem at different times, so that every time, applying the new knowledge to reduce the cost of resolving. The problem mentioned they can solve it before teaching the Pythagorean theorem with cost 10, after teaching the Pythagorean theorem with minimal cost 7, while after and teaching the trigonometric numbers with minimal cost 6. We believe that this teaching practice contributes to connection the old with the new knowledge. It helps the students to see the one as a continuation of another, and both of them as parts of a single mathematical wholeness rather as piecemeal and uncorrelated between them.
- The proposed problems enable us, with appropriate change in access costs, to diversify optimal resolving strategy depending to the learning target. About the problem we mentioned, we can, with appropriate change in costs, make as optimal strategy the use only the Pythagorean Theorem or only trigonometric numbers or any one combination thereof.
- The assignment of selecting data on the students themselves, it creates a different kind of relationship between the student and the path to the solution. The path to a solution is not determined by the problem itself, but the student through data selection, actively involved in setting it. The question is whether this framework creates interest incentives if enhances the will of the student to his involvement with mathematics.
- Many resolving strategies the students developed when dealing with a proposed problem, many different problems partially construct and solve. The construction and resolve out simultaneously, resulting to not treat them as two distinct processes as usual. They grapple not only with the one, the solution, but also with both, as interdependent own actions.
- The multiple ways to solve classified cognitive level, allow tackling the problem from a wide range of students. The question is whether this framework can work positively to students who demonstrate a systematic failing and aversion to mathematics, to see in a different way not only the math but also their abilities in math.
- A perspective and an alternative form of problems with data option is the optional data not to be predetermined from the outset, but to be seek out and collected by the students themselves without any restrictions.
- The proposed problems appear to be closer to nature of real problems. In a real problem, always the question is asked, but never which data we will use to face it.

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