ΑΠΟΛΥΤΗΡΙΕΣ ΕΞΕΤΑΣΕΙΣ Δ΄ ΤΑΞΗΣ
ΕΣΠΕΡΙΝΟΥ ΕΝΙΑΙΟΥ ΛΥΚΕΙΟΥ
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ΑΠΑΝΤΗΣΕΙΣ ΣΤΑ ΜΑΘΗΜΑΤΙΚΑ ΘΕΤΙΚΗΣ ΚΑΤΕΥΘΥΝΣΗΣ

ΘΕΜΑ 1^{ο}

Α.α. Σχολικό βιβλίο σελίδα 30.

β. 1. Δ, **2.** Α

Β.α. $|A| = \begin{vmatrix} 2 & -3 \\ -2 & 4 \end{vmatrix} = 8 - 6 = 2 \neq 0$

$$A^{-1} = \frac{1}{2}\begin{bmatrix} 4 & 3 \\ 2 & 2 \end{bmatrix} = \begin{bmatrix} 2 & \frac{3}{2} \\ 1 & 1 \end{bmatrix}$$

β. $AX = B \Leftrightarrow X = A^{-1}\cdot B \Leftrightarrow X = \begin{bmatrix} 2 & \frac{3}{2} \\ 1 & 1 \end{bmatrix}\cdot\begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} \frac{5}{2} & \frac{3}{2} \\ 2 & 1 \end{bmatrix}$.

ΘΕΜΑ 2^{ο}

α. $z_1 \cdot z_2 = (7 + 8i)(4 - 5i) = 28 - 35i + 32i + 40 = \mathbf{68 - 3i}$

β. $\frac{z_1}{z_2} = \frac{7 + 8i}{4 - 5i} = \frac{(7 + 8i)(4 + 5i)}{(4 - 5i)(4 + 5i)} = \frac{28 + 35i + 32i - 40}{16 + 25} = \frac{-12 + 67i}{41}$

$= \mathbf{-\frac{12}{41} + \frac{67}{41}i}$

γ. $z = z_1 - \overline{z_2} = 7 + 8i - (4 + 5i) = 7 + 8i - 4 - 5i = 3 + 3i$

$\rho = |z| = |3 + 3i| = \sqrt{3^2 + 3^2} = \sqrt{18} = 3\sqrt{2}$

$$\left.\begin{aligned} συνφ &= \frac{α}{ρ} = \frac{3}{3\sqrt{2}} = \frac{\sqrt{2}}{2} \\ ημφ &= \frac{β}{ρ} = \frac{3}{3\sqrt{2}} = \frac{\sqrt{2}}{2} \end{aligned}\right\} \Rightarrow φ = \frac{π}{4}$$

$z = 3\sqrt{2}\left(συν\frac{π}{4} + iημ\frac{π}{4}\right)$

$$z^4 = \left[3\sqrt{2}\left(\sigma\upsilon\nu\frac{\pi}{4} + i\eta\mu\frac{\pi}{4}\right)\right]^4 = \left(3\sqrt{2}\right)^4\left(\sigma\upsilon\nu\frac{\pi}{4} + i\eta\mu\frac{\pi}{4}\right)^4$$

$$= 324\left(\sigma\upsilon\nu\frac{4\pi}{4} + i\eta\mu\frac{4\pi}{4}\right) = 324\left(\sigma\upsilon\nu\pi + i\eta\mu\pi\right) = 324(-1 + 0i) = \mathbf{-324}$$

ή $z^4 = (3 + 3i)^4 = \left[3(1 + i)\right]^4 = 3^4(1 + i)^4 = 81\left[(1 + i)^2\right]^2$

$$= 81(1 + 2i - 1)^2 = 81(2i)^2 = 81(-4) = \mathbf{-324}$$

ΘΕΜΑ 3ο

α. $\lim\limits_{x\to1^-} f(x) = \lim\limits_{x\to1^-}\dfrac{x^2 - x}{x - 1} = \lim\limits_{x\to1^-}\dfrac{x\cancel{(x-1)}}{\cancel{x-1}} = \lim\limits_{x\to1^-} x = 1$

$\lim\limits_{x\to1^+} f(x) = \lim\limits_{x\to1^+}(\alpha x - 2\alpha + 3) = \alpha - 2\alpha + 3 = 3 - \alpha$

$f(1) = \alpha - 2\alpha + 3 = 3 - \alpha$

$\overset{f \text{ συνεχής}}{\underset{\text{στο } x_0 = 1}{\Rightarrow}}$

$3 - \alpha = 1 \Leftrightarrow \mathbf{\alpha = 2}$

β. $\lim\limits_{x\to-2} f(x) = \lim\limits_{x\to-2}\dfrac{x^2 - x}{x - 1} = \dfrac{(-2)^2 - (-2)}{-2 - 1} = \dfrac{4 + 2}{-3} = \dfrac{6}{-3} = \mathbf{-2}$

$\lim\limits_{x\to2} f(x) = \lim\limits_{x\to2}(\alpha x - 2\alpha + 3) = 2\alpha - 2\alpha + 3 = \mathbf{3}$

ΘΕΜΑ 4ο

α. $f'(x) = \left(\dfrac{1}{9}x^3 - \dfrac{1}{3}x^2 - x + 10\right)' = \dfrac{1}{3}x^2 - \dfrac{2}{3}x - 1,\ 1 < x < 5$

$f'(x) = 0 \Leftrightarrow \dfrac{1}{3}x^2 - \dfrac{2}{3}x - 1 = 0 \Leftrightarrow x^2 - 2x - 3 = 0 \Leftrightarrow x = 3$ ή $\cancel{x = -1}$

x	1		3		5
f΄(x)		-	0	+	
f (x)		↘		↗	

Η κατανάλωση γίνεται ελάχιστη όταν **x = 3**

Ελάχιστη κατανάλωση : $f_{min} = f(3) = \frac{1}{9}3^3 - \frac{1}{3}3^2 - 3 + 10 = \mathbf{7}$

β. $f'(2) = \frac{1}{3}2^2 - \frac{2}{3}2 - 1 = \frac{4}{3} - \frac{4}{3} - 1 = \mathbf{-1}$

$f'(4) = \frac{1}{3}4^2 - \frac{2}{3}4 - 1 = \frac{16}{3} - \frac{8}{3} - 1 = \frac{8}{3} - 1 = \mathbf{\frac{5}{3}}$