CHAPTER 5

Useful Circuit Analysis Techniques

INTRODUCTION

The techniques of nodal and mesh analysis described in Chap. 4 are reliable and extremely powerful methods. However, both require that we develop a complete set of equations to describe a particular circuit as a general rule, even if only one current, voltage, or power quantity is of interest. In this chapter, we investigate several different techniques for isolating specific parts of a circuit in order to simplify the analysis. After examining the usage of these techniques, we focus on how one might go about selecting one method over another.

5.1 LINEARITY AND SUPERPOSITION

All of the circuits which we plan to analyze can be classified as *linear circuits,* so this is a good time to be more specific in defining exactly what we mean by that. Having done this, we can then consider the most important consequence of linearity, the principle of ***superposition.*** This principle is very basic and will appear repeatedly in our study of linear circuit analysis. As a matter of fact, the nonapplicability of superposition to nonlinear circuits is the very reason they are so difficult to analyze!

> The principle of superposition states that the *response* (a desired current or voltage) in a linear circuit having more than one independent source can be obtained by adding the responses caused by the separate independent sources *acting alone*.

Linear Elements and Linear Circuits

Let us first define a ***linear element*** as a passive element that has a linear voltage-current relationship. By a "linear voltage-current

KEY CONCEPTS

- Superposition as a Means of Determining the *Individual Contributions* of Different Sources to Any Current or Voltage
- Source Transformation as a Means of Simplifying Circuits
- Thévenin's Theorem
- Norton's Theorem
- Thévenin and Norton Equivalent Networks
- Maximum Power Transfer
- $\Delta \leftrightarrow$ Y Transformations for Resistive Networks
- Selecting a Particular Combination of Analysis Techniques
- Performing dc Sweep Simulations Using PSpice

relationship" we simply mean that multiplication of the current through the element by a constant K results in the multiplication of the voltage across the element by the same constant K. At this time, only one passive element has been defined (the resistor) and its voltage-current relationship

$$v(t) = Ri(t)$$

is clearly linear. As a matter of fact, if $v(t)$ is plotted as a function of $i(t)$, the result is a straight line.

We must also define a ***linear dependent source*** as a dependent current or voltage source whose output current or voltage is proportional only to the first power of a specified current *or* voltage variable in the circuit (or to the *sum* of such quantities). For example, the dependent voltage source $v_s = 0.6i_1 - 14v_2$ is linear, but $v_s = 0.6i_1^2$ and $v_s = 0.6i_1v_2$ are not.

We may now define a ***linear circuit*** as a circuit composed entirely of independent sources, linear dependent sources, and linear elements. From this definition, it is possible to show[1] that "the response is proportional to the source," or that multiplication of all independent source voltages and currents by a constant K increases all the current and voltage responses by the same factor K (including the dependent source voltage or current outputs).

The Superposition Principle

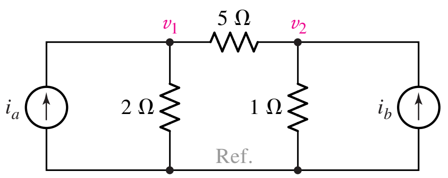


FIGURE 5.1 A circuit with two independent current sources.

The most important consequence of linearity is superposition. Let us develop the superposition principle by considering first the circuit of Fig. 5.1, which contains two independent sources, the current generators that force the currents i_a and i_b into the circuit. Sources are often called *forcing functions* for this reason, and the nodal voltages that they produce can be termed *response functions,* or simply *responses*. Both the forcing functions and the responses may be functions of time. The two nodal equations for this circuit are

$$0.7v_1 - 0.2v_2 = i_a \quad [1]$$

$$-0.2v_1 + 1.2v_2 = i_b \quad [2]$$

Now let us perform experiment x. We change the two forcing functions to i_{ax} and i_{bx}; the two unknown voltages will now be different, so we will call them v_{1x} and v_{2x}. Thus,

$$0.7v_{1x} - 0.2v_{2x} = i_{ax} \quad [3]$$

$$-0.2v_{1x} + 1.2v_{2x} = i_{bx} \quad [4]$$

We next perform experiment y by changing the source currents to i_{ay} and i_{by} and measure the responses v_{1y} and v_{2y}:

$$0.7v_{1y} - 0.2v_{2y} = i_{ay} \quad [5]$$

$$-0.2v_{1y} + 1.2v_{2y} = i_{by} \quad [6]$$

(1) The proof involves first showing that the use of nodal analysis on the linear circuit can produce only linear equations of the form

$$a_1v_1 + a_2v_2 + \cdots + a_Nv_N = b$$

where the a_i are constants (combinations of resistance or conductance values, constants appearing in dependent source expressions, 0, or ± 1), the v_i are the unknown node voltages (responses), and b is an independent source value or a sum of independent source values. Given a set of such equations, if we multiply all the b's by K, then it is evident that the solution of this new set of equations will be the node voltages $Kv_1, Kv_2, \ldots, Kv_N$.

These three sets of equations describe the same circuit with three different sets of source currents. Let us *add* or "*superpose*" the last two sets of equations. Adding Eqs. [3] and [5],

$$(0.7v_{1x} + 0.7v_{1y}) - (0.2v_{2x} + 0.2v_{2y}) = i_{ax} + i_{ay} \qquad [7]$$

$$0.7v_1 \qquad - \qquad 0.2v_2 \qquad = \qquad i_a \qquad [1]$$

and adding Eqs. [4] and [6],

$$-(0.2v_{1x} + 0.2v_{1y}) + (1.2v_{2x} + 1.2v_{2y}) = i_{bx} + i_{by} \qquad [8]$$

$$-0.2v_1 \qquad + \qquad 1.2v_2 \qquad = \qquad i_b \qquad [2]$$

where Eq. [1] has been written immediately below Eq. [7] and Eq. [2] below Eq. [8] for easy comparison.

The linearity of all these equations allows us to compare Eq. [7] with Eq. [1] and Eq. [8] with Eq. [2] and draw an interesting conclusion. If we select i_{ax} and i_{ay} such that their sum is i_a and select i_{bx} and i_{by} such that their sum is i_b, then the desired responses v_1 and v_2 may be found by adding v_{1x} to v_{1y} and v_{2x} to v_{2y}, respectively. In other words, we can perform experiment x and note the responses, perform experiment y and note the responses, and finally add the two sets of responses. This leads to the fundamental concept involved in the superposition principle: to look at each independent source (and the response it generates) one at a time with the other independent sources "turned off" or "zeroed out."

If we reduce a voltage source to zero volts, we have effectively created a short circuit (Fig. 5.2*a*). If we reduce a current source to zero amps, we have effectively created an open circuit (Fig. 5.2*b*). Thus, the ***superposition theorem*** can be stated as:

> In any linear resistive network, the voltage across or the current through any resistor or source may be calculated by adding algebraically all the individual voltages or currents caused by the separate independent sources acting alone, with all other independent voltage sources replaced by short circuits and all other independent current sources replaced by open circuits.

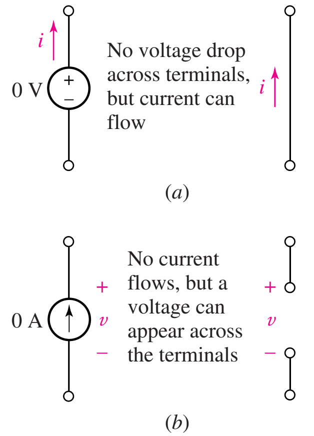


FIGURE 5.2 (*a*) A voltage source set to zero acts like a short circuit. (*b*) A current source set to zero acts like an open circuit.

Thus, if there are N independent sources we must perform N experiments, each having only one of the independent sources active and the others inactive/turned off/zeroed out. Note that *dependent* sources are in general active in every experiment.

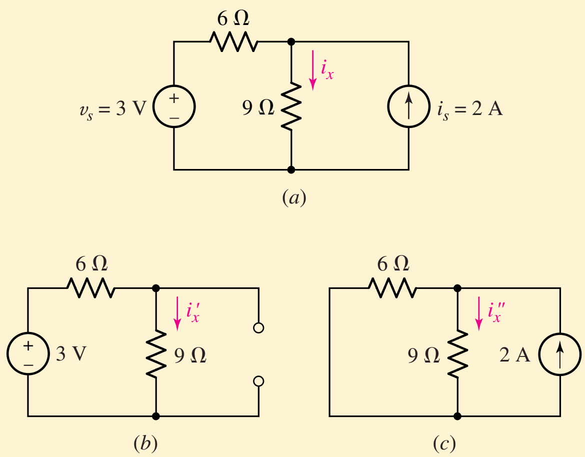
The circuit we have just used as an example, however, should indicate that a much stronger theorem might be written; a *group* of independent sources may be made active and inactive collectively, if we wish. For example, suppose there are three independent sources. The theorem states that we may find a given response by considering each of the three sources acting alone and adding the three results. Alternatively, we may find the response due to the first and second sources operating with the third inactive, and then add to this the response caused by the third source acting alone. This amounts to treating several sources collectively as a sort of "supersource."

There is also no reason that an independent source must assume only its given value or a zero value in the several experiments; it is necessary only for the sum of the several values to be equal to the original value. An inactive source almost always leads to the simplest circuit, however.

Let us illustrate the application of the superposition principle by considering an example in which both types of independent source are present.

EXAMPLE 5.1

For the circuit of Fig. 5.3*a*, use superposition to write an expression for the unknown branch current i_x.



■ **FIGURE 5.3** (*a*) An example circuit with two independent sources for which the branch current i_x is desired; (*b*) same circuit with current source open-circuited; (*c*) original circuit with voltage source short-circuited.

We first set the current source equal to zero and redraw the circuit as shown in Fig. 5.3*b*. The portion of i_x due to the voltage source has been designated i_x' to avoid confusion and is easily found to be 0.2 A.

We next set the voltage source in Fig. 5.3*a* to zero and again redraw the circuit, as shown in Fig. 5.3*c*. Routine application of current division allows us to determine that i_x'' (the portion of i_x due to the 2 A current source) is 0.8 A.

We may now compute the complete current i_x as the sum of the two individual components:

$$i_x = i_x|_{3\text{ V}} + i_x|_{2\text{ A}} = i_x' + i_x''$$

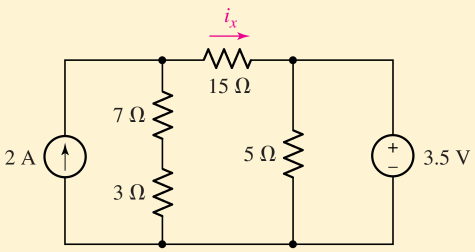
or

$$i_x = \frac{3}{6+9} + 2\left(\frac{6}{6+9}\right) = 0.2 + 0.8 = 1.0\text{ A}$$

Another way of looking at Example 5.1 is that the 3 V source and the 2 A source are each performing work on the circuit, resulting in a total current i_x flowing through the 9 Ω resistor. *However, the contribution of the 3 V source to i_x does not depend on the contribution of the 2 A source, and vice versa.* For example, if we double the output of the 2 A source to 4 A, it will now contribute 1.6 A to the total current i_x flowing through the 9 Ω resistor. However, the 3 V source would still contribute only 0.2 A to i_x, for a new total current of 0.2 + 1.6 = 1.8 A.

PRACTICE

5.1 For the circuit of Fig. 5.4, use superposition to compute the current i_x.



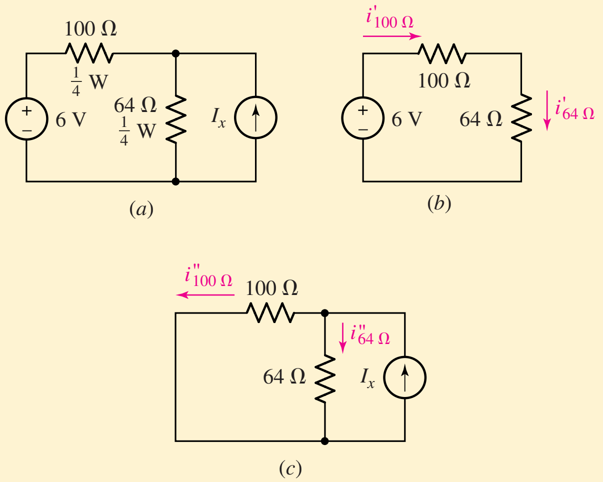
■ **FIGURE 5.4**

Ans: 660 mA.

As we will see, superposition does not generally reduce our workload when considering a particular circuit, since it leads to the analysis of several new circuits to obtain the desired response. However, it is particularly useful in identifying the significance of various parts of a more complex circuit. It also forms the basis of phasor analysis, which is introduced in Chap. 10.

EXAMPLE 5.2

Referring to the circuit of Fig. 5.5*a*, determine the maximum *positive* current to which the source I_X can be set before any resistor exceeds its power rating and overheats.



■ **FIGURE 5.5** (*a*) A circuit with two resistors each rated at $\frac{1}{4}$ W. (*b*) Circuit with only the 6 V source active. (*c*) Circuit with the source I_x active.

▶ ***Identify the goal of the problem.***

Each resistor is rated to a maximum of 250 mW. If the circuit allows this value to be exceeded (by forcing too much current through either resistor), excessive heating will occur—possibly leading to

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an accident. The 6 V source cannot be changed, so we are looking for an equation involving I_x and the maximum current through each resistor.

Collect the known information.

Based on its 250 mW power rating, the maximum current the 100 Ω resistor can tolerate is

$$\sqrt{\frac{P_{max}}{R}} = \sqrt{\frac{0.250}{100}} = 50 \text{ mA}$$

and, similarly, the current through the 64 Ω resistor must be less than 62.5 mA.

Devise a plan.

Either nodal or mesh analysis may be applied to the solution of this problem, but superposition may give us a slight edge, since we are primarily interested in the effect of the current source.

Construct an appropriate set of equations.

Using superposition, we redraw the circuit as in Fig. 5.5*b* and find that the 6 V source contributes a current

$$i'_{100\Omega} = \frac{6}{100 + 64} = 36.59 \text{ mA}$$

to the 100 Ω resistor and, since the 64 Ω resistor is in series, $i'_{64\Omega} =$ 36.59 mA as well.

Recognizing the current divider in Fig. 5.5*c*, we note that $i''_{64\Omega}$ will *add* to $i'_{64\Omega}$, but $i''_{100\Omega}$ is *opposite* in direction to $i'_{100\Omega}$. I_X can therefore safely contribute $62.5 - 36.59 = 25.91$ mA to the 64 Ω resistor current, and $50 - (-36.59) = 86.59$ mA to the 100 Ω resistor current.

The 100 Ω resistor therefore places the following constraint on I_X:

$$I_X < (86.59 \times 10^{-3})\left(\frac{100 + 64}{64}\right)$$

and the 64 Ω resistor requires that

$$I_X < (25.91 \times 10^{-3})\left(\frac{100 + 64}{100}\right)$$

Attempt a solution.

Considering the 100 Ω resistor first, we see that I_X is limited to $I_X <$ 221.9 mA. The 64 Ω resistor limits I_X such that $I_X <$ 42.49 mA.

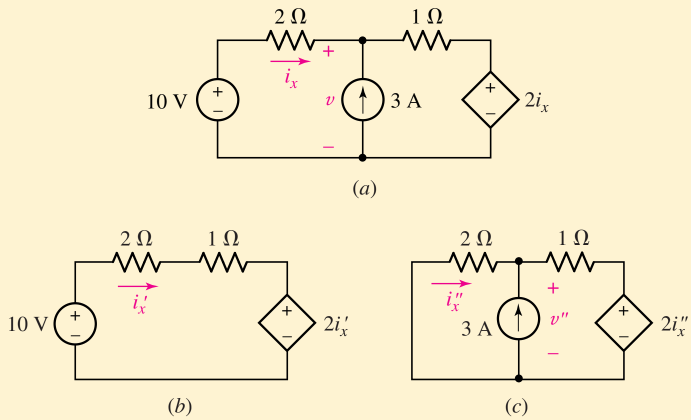
Verify the solution. Is it reasonable or expected?

In order to satisfy both constraints, I_X must be less than 42.49 mA. If the value is increased, the 64 Ω resistor will overheat long before the 100 Ω resistor does. One particularly useful way to evaluate our solution is to perform a dc sweep analysis in PSpice as described after the next example. An interesting question, however, is whether we would have expected the 64 Ω resistor to overheat first.

Originally we found that the 100 Ω resistor has a smaller maximum current, so it might be reasonable to expect it to limit I_X. However, because I_X *opposes* the current sent by the 6 V source through the 100 Ω resistor but *adds* to the 6 V source's contribution to the current through the 64 Ω resistor, it turns out to work the other way—it's the 64 Ω resistor that sets the limit on I_X.

EXAMPLE 5.3

In the circuit of Fig. 5.6*a*, use the superposition principle to determine the value of i_x.



■ **FIGURE 5.6** (*a*) An example circuit with two independent sources and one dependent source for which the branch current i_x is desired. (*b*) Circuit with the 3 A source open-circuited. (*c*) Original circuit with the 10 V source short-circuited.

We first open-circuit the 3 A source (Fig. 5.6*b*). The single mesh equation is

$$-10 + 2i'_x + 1i'_x + 2i'_x = 0$$

so that

$$i'_x = 2 \text{ A}$$

Next, we short-circuit the 10 V source (Fig. 5.6*c*) and write the single-node equation

$$\frac{v''}{2} + \frac{v'' - 2i''_x}{1} = 3$$

and relate the dependent-source-controlling quantity to v'':

$$v'' = 2(-i''_x)$$

We find

$$i''_x = -0.6 \text{ A}$$

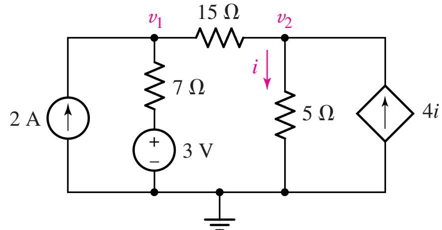
and, thus,

$$i_x = i'_x + i''_x = 2 + (-0.6) = 1.4 \text{ A}$$

Note that in redrawing each subcircuit, we are always careful to use some type of notation to indicate that we are not working with the original variables. This prevents the possibility of rather disastrous errors when we add the individual results.

PRACTICE

5.2 For the circuit of Fig. 5.7, use superposition to obtain the voltage across each current source.



■ **FIGURE 5.7**

Ans: $v_1|_{2A} = 9.180$ V, $v_2|_{2A} = -1.148$ V, $v_1|_{3V} = 1.967$ V, $v_2|_{3V} = -0.246$ V; $v_1 = 11.147$ V, $v_2 = -1.394$ V.

Summary of Basic Superposition Procedure

1. **Select one of the independent sources. Set all other independent sources to zero.** This means voltage sources are replaced with short circuits and current sources are replaced with open circuits. Leave dependent sources alone.
2. **Relabel voltages and currents using suitable notation** (e.g., v', i_2''). Be sure to relabel controlling variables of dependent sources to avoid confusion.
3. **Analyze the simplified circuit to find the desired currents and/or voltages.**
4. **Repeat steps 1 through 3 until each independent source has been considered.**
5. **Add the partial currents and/or voltages obtained from the separate analyses.** Pay careful attention to voltage signs and current directions when summing.
6. **Do not add power quantities.** If power quantities are required, calculate only after partial voltages and/or currents have been summed.

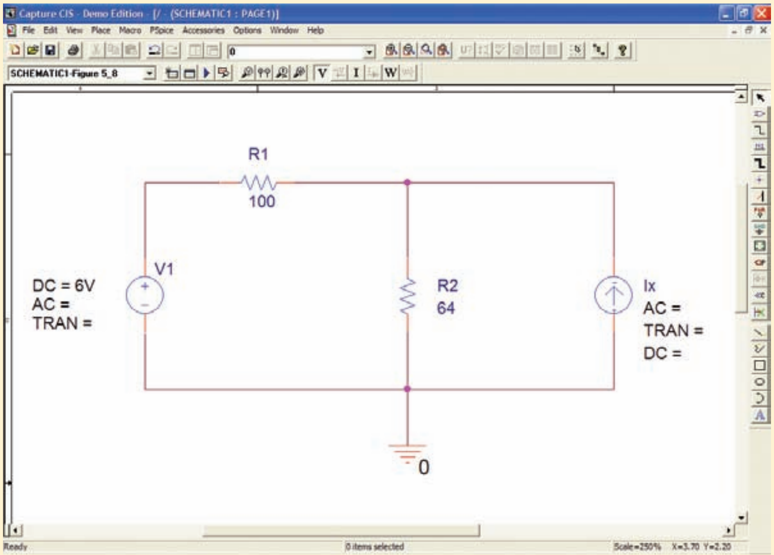
Note that step 1 may be altered in several ways. First, independent sources can be considered in groups as opposed to individually if it simplifies the analysis, as long as no independent source is included in more than one subcircuit. Second, it is technically not necessary to set sources to zero, although this is almost always the best route. For example, a 3 V source may appear in two subcircuits as a 1.5 V source, since $1.5 + 1.5 = 3$ V just as $0 + 3 = 3$ V. Because it is unlikely to simplify our analysis, however, there is little point to such an exercise.

COMPUTER-AIDED ANALYSIS

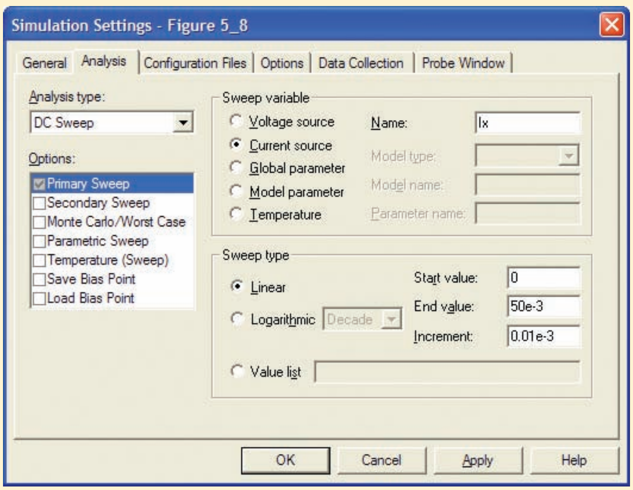
Although PSpice is extremely useful in verifying that we have analyzed a complete circuit correctly, it can also assist us in determining the contribution of each source to a particular response. To do this, we employ what is known as a *dc parameter sweep*.

Consider the circuit presented in Example 5.2, when we were asked to determine the maximum positive current that could be obtained from the current source without exceeding the power rating of either resistor in the circuit. The circuit is shown redrawn using the Orcad Capture CIS schematic tool in Fig. 5.8. Note that no value has been assigned to the current source.

After the schematic has been entered and saved, the next step is to specify the dc sweep parameters. This option allows us to specify a range of values for a voltage or current source (in the present case, the current source I_x), rather than a specific value. Selecting **New Simulation Profile** under **PSpice,** we provide a name for our profile and are then provided with the dialog box shown in Fig. 5.9.



■ **FIGURE 5.8** The circuit from Example 5.2.



■ **FIGURE 5.9** DC Sweep dialog box shown with I_x selected as the sweep variable.

Under **Analysis Type,** we pull down the **DC Sweep** option, specify the "sweep variable" as **Current Source,** and then type in I_x in the **Name** box. There are several options under Sweep Type: **Linear, Logarithmic,** and **Value List.** The last option allows us to specify each value to assign to I_x. In order to generate a smooth plot, however, we choose to perform a **Linear** sweep, with a **Start Value** of 0 mA, an **End Value** of 50 mA, and a value of 0.01 mA for the **Increment.**

After we perform the simulation, the graphical output package Probe is automatically launched. When the window appears, the horizontal axis (corresponding to our variable, I_x) is displayed, but the vertical axis variable must be chosen. Selecting **Add Trace** from the **Trace**

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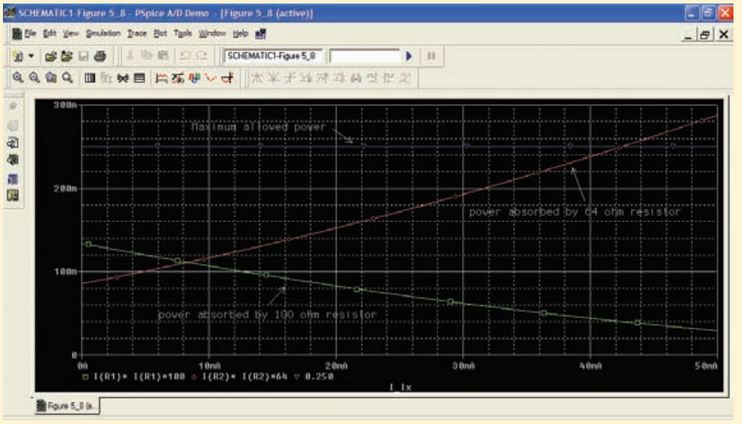


(*a*)

(*b*)

FIGURE 5.10 (*a*) Probe output with text labels identifying the power absorbed by the two resistors individually. A horizontal line indicating 250 mW has also been included, as well as text labels to improve clarity. (*b*) Cursor dialog box.

menu, we click on **I(R1),** then type an asterisk in the **Trace Expression** box, click on **I(R1)** once again, insert yet another asterisk, and finally type in 100. This asks Probe to plot the power absorbed by the 100 Ω resistor. In a similar fashion, we repeat the process to add the power absorbed by the 64 Ω resistor, resulting in a plot similar to that shown in Fig. 5.10*a*. A horizontal reference line at 250 mW was also added to the plot by typing 0.250 in the **Trace Expression** box after selecting **Add Trace** from the **Trace** menu a third time.

We see from the plot that the 64 Ω resistor *does* exceed its 250 mW power rating in the vicinity of $I_x = 43$ mA. In contrast, however, we see that regardless of the value of the current source I_x (provided that it is between 0 and 50 mA), the 100 Ω resistor will never dissipate 250 mW; in fact, the absorbed power *decreases* with increasing current from the current source. If we desire a more precise answer we can make use of the cursor tool, which is invoked by selecting **Trace, Cursor, Display** from the menu bar. Figure 5.10*b* shows the result of dragging both cursors to 42.53 mA; the 64 Ω resistor has just barely exceeded its rating at this current level. Increased precision can be obtained by decreasing the increment value used in the dc sweep.

This technique is very useful in analyzing electronic circuits, where we might need, for example, to determine what input voltage is required to a complicated amplifier circuit in order to obtain a zero output voltage. We also notice that there are several other types of parameter sweeps that we can perform, including a dc voltage sweep. The ability to vary temperature is useful only when dealing with component models that have a temperature parameter built in, such as diodes and transistors.

Unfortunately, it usually turns out that little if any time is saved in analyzing a circuit containing one or more dependent sources by use of the superposition principle, for there must always be at least two sources in operation: one independent source and all the dependent sources.

We must constantly be aware of the limitations of superposition. It is applicable only to linear responses, and thus the most common nonlinear response—power—is not subject to superposition. For example, consider two 1 V batteries in series with a 1 Ω resistor. The power delivered to the resistor is obviously 4 W, but if we mistakenly try to apply superposition we might say that each battery alone furnished 1 W and thus the total power is 2 W. This is incorrect, but a surprisingly easy mistake to make.

5.2 SOURCE TRANSFORMATIONS

Practical Voltage Sources

Up to now we have been working exclusively with *ideal* voltage and current sources; it is now time to take a step closer to reality by considering *practical* sources. These sources will enable us to make more realistic representations of physical devices. Once we have defined practical sources, we will see that *practical* current and voltage sources may be interchanged without affecting the remainder of the circuit. Such sources will be called *equivalent* sources. Our methods will be applicable to both independent and dependent sources, although we will find that they are not as useful with dependent sources.

The ideal voltage source was defined as a device whose terminal voltage is independent of the current through it. A 1 V dc source produces a current of 1 A through a 1 Ω resistor, and a current of 1,000,000 A through a 1 $\mu\Omega$ resistor; it can provide an unlimited amount of power. No such device exists practically, of course, and we agreed previously that a *real* physical voltage source could be represented by an *ideal* voltage source only as long as relatively small currents, or powers, were drawn from it. For example, a car battery may be approximated by an ideal 12 V dc voltage source if its current is limited to a few amperes (Fig. 5.11*a*). However, anyone who has ever tried to start an automobile with the headlights on must have observed that the lights dimmed perceptibly when the battery was asked to deliver the heavy starter current, 100 A or more, in addition to the headlight current. Under these conditions, an ideal voltage source is not really an adequate representation of the battery.

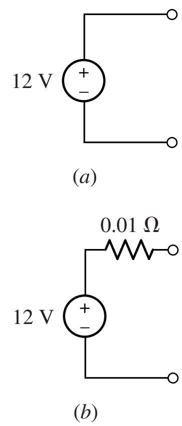
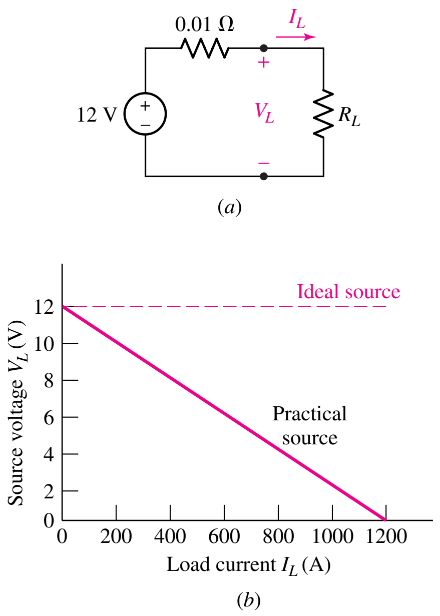
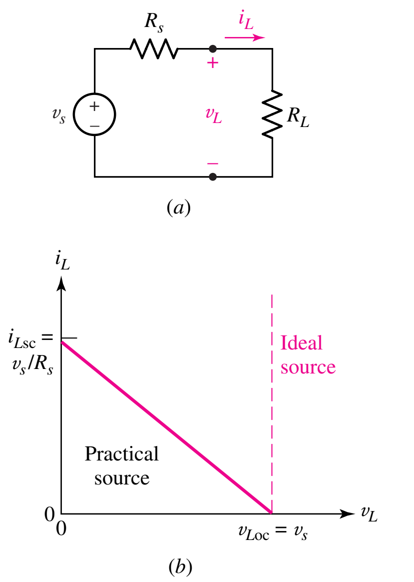


FIGURE 5.11 (*a*) An ideal 12 V dc voltage source used to model a car battery. (*b*) A more accurate model that accounts for the observed reduction in terminal voltage at large currents.

To better approximate the behavior of a real device, the ideal voltage source must be modified to account for the lowering of its terminal voltage when large currents are drawn from it. Let us suppose that we observe experimentally that our car battery has a terminal voltage of 12 V when no current is flowing through it, and a reduced voltage of 11 V when 100 A is flowing. How could we model this behavior? Well, a more accurate model might be an ideal voltage source of 12 V in series with a resistor across which 1 V appears when 100 A flows through it. A quick calculation shows that the resistor must be 1 V/100 A = 0.01 Ω, and the ideal voltage source and this series resistor constitute a ***practical voltage source*** (Fig. 5.11*b*). Thus, we are using the series combination of two ideal circuit elements, an independent voltage source and a resistor, to model a real device.



■ **FIGURE 5.12** (*a*) A practical source, which approximates the behavior of a certain 12 V automobile battery, is shown connected to a load resistor R_L. (*b*) The relationship between I_L and V_L is linear.



■ **FIGURE 5.13** (*a*) A general practical voltage source connected to a load resistor R_L. (*b*) The terminal voltage of a practical voltage source decreases as i_L increases and $R_L = v_L/i_L$ decreases. The terminal voltage of an ideal voltage source (also plotted) remains the same for any current delivered to a load.

We do not expect to find such an arrangement of ideal elements inside our car battery, of course. Any real device is characterized by a certain current-voltage relationship at its terminals, and our problem is to develop some combination of ideal elements that can furnish a similar current-voltage characteristic, at least over some useful range of current, voltage, or power.

In Fig. 5.12*a*, we show our two-piece practical model of the car battery now connected to some load resistor R_L. The terminal voltage of the practical source is the same as the voltage across R_L and is marked[2] V_L. Figure 5.12*b* shows a plot of load voltage V_L as a function of the load current I_L for this practical source. The KVL equation for the circuit of Fig. 5.12*a* may be written in terms of I_L and V_L:

$$12 = 0.01 I_L + V_L$$

and thus

$$V_L = -0.01 I_L + 12$$

This is a linear equation in I_L and V_L, and the plot in Fig. 5.12*b* is a straight line. Each point on the line corresponds to a different value of R_L. For example, the midpoint of the straight line is obtained when the load resistance is equal to the internal resistance of the practical source, or $R_L = 0.01\ \Omega$. Here, the load voltage is exactly one-half the ideal source voltage.

When $R_L = \infty$ and no current whatsoever is being drawn by the load, the practical source is open-circuited and the terminal voltage, or open-circuit voltage, is $V_{Loc} = 12$ V. If, on the other hand, $R_L = 0$, thereby short-circuiting the load terminals, then a load current or short-circuit current, $I_{Lsc} = 1200$ A, would flow. (*In practice, such an experiment would probably result in the destruction of the short circuit, the battery, and any measuring instruments incorporated in the circuit!*)

Since the plot of V_L versus I_L is a straight line for this practical voltage source, we should note that the values of V_{Loc} and I_{Lsc} uniquely determine the entire $V_L - I_L$ curve.

The horizontal broken line of Fig. 5.12*b* represents the $V_L - I_L$ plot for an *ideal* voltage source; the terminal voltage remains constant for any value of load current. For the practical voltage source, the terminal voltage has a value near that of the ideal source only when the load current is relatively small.

Let us now consider a *general* practical voltage source, as shown in Fig. 5.13*a*. The voltage of the ideal source is v_s, and a resistance R_s, called an *internal resistance* or *output resistance,* is placed in series with it. Again, we must note that the resistor is not really present as a separate component but merely serves to account for a terminal voltage that decreases as the load current increases. Its presence enables us to model the behavior of a physical voltage source more closely.

The linear relationship between v_L and i_L is

$$v_L = v_s - R_s i_L \qquad [9]$$

(2) From this point on we will endeavor to adhere to the standard convention of referring to strictly dc quantities using capital letters, whereas lowercase letters denote a quantity that we know to possess some time-varying component. However, in describing general theorems which apply to either dc or ac, we will continue to use lowercase to emphasize the general nature of the concept.

and this is plotted in Fig. 5.13*b*. The open-circuit voltage ($R_L = \infty$, so $i_L = 0$) is

$$v_{Loc} = v_s \qquad [10]$$

and the short-circuit current ($R_L = 0$, so $v_L = 0$) is

$$i_{Lsc} = \frac{v_s}{R_s} \qquad [11]$$

Once again, these values are the intercepts for the straight line in Fig. 5.13*b*, and they serve to define it completely.

Practical Current Sources

An ideal current source is also nonexistent in the real world; there is no physical device that will deliver a constant current regardless of the load resistance to which it is connected or the voltage across its terminals. Certain transistor circuits will deliver a constant current to a wide range of load resistances, but the load resistance can always be made sufficiently large that the current through it becomes very small. Infinite power is simply never available (unfortunately).

A practical current source is defined as an ideal current source in parallel with an internal resistance R_p. Such a source is shown in Fig. 5.14*a*, and the current i_L and voltage v_L associated with a load resistance R_L are indicated. Application of KCL yields

$$i_L = i_s - \frac{v_L}{R_p} \qquad [12]$$

which is again a linear relationship. The open-circuit voltage and the short-circuit current are

$$v_{Loc} = R_p i_s \qquad [13]$$

and

$$i_{Lsc} = i_s \qquad [14]$$

The variation of load current with changing load voltage may be investigated by changing the value of R_L as shown in Fig. 5.14*b*. The straight line is traversed from the short-circuit, or "northwest," end to the open-circuit termination at the "southeast" end by increasing R_L from zero to infinite ohms. The midpoint occurs for $R_L = R_p$. The load current i_L and the ideal source current are approximately equal only for small values of load voltage, which are obtained with values of R_L that are small compared to R_p.

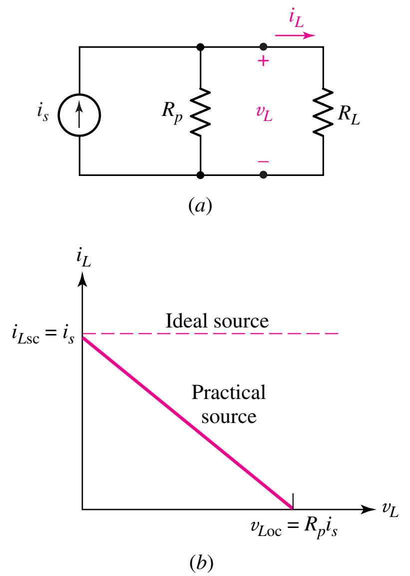


FIGURE 5.14 (*a*) A general practical current source connected to a load resistor R_L. (*b*) The load current provided by the practical current source is shown as a function of the load voltage.

Equivalent Practical Sources

Having defined both practical sources, we are now ready to discuss their equivalence. We will define two sources as being *equivalent* if they produce identical values of v_L and i_L when they are connected to identical values of R_L, no matter what the value of R_L may be. Since $R_L = \infty$ and $R_L = 0$ are two such values, equivalent sources provide the same open-circuit voltage and short-circuit current. In other words, if we are given two equivalent sources, one a practical voltage source and the other a practical current source, each enclosed in a black box with only a single pair of terminals,

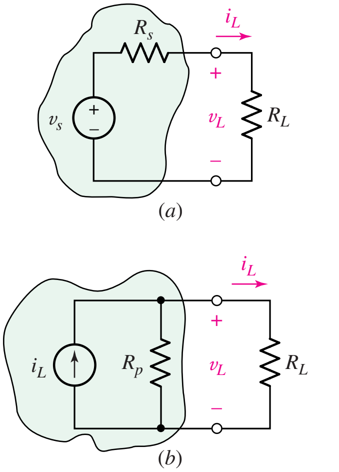


FIGURE 5.15 (*a*) A given practical voltage source connected to a load R_L. (*b*) The equivalent practical current source connected to the same load.

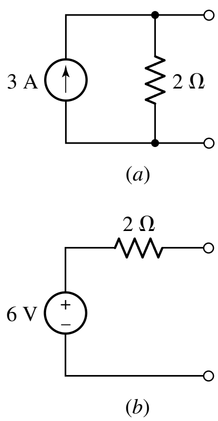


FIGURE 5.16 (*a*) A given practical current source. (*b*) The equivalent practical voltage source.

then there is no way in which we can tell which source is in which box by measuring current or voltage in a resistive load.

Consider the practical voltage source and resistor R_L shown in Fig. 5.15*a*, and the circuit composed of a practical current source and resistor R_L shown in Fig. 5.15*b*. A simple calculation shows that the voltage across the load R_L of Fig. 5.15*a* is

$$v_L = v_s \frac{R_L}{R_s + R_L} \qquad [15]$$

A similarly simple calculation shows that the voltage across the load R_L in Fig. 5.15*b* is

$$v_L = \left[i_s \frac{R_p}{R_p + R_L} \right] \cdot R_L$$

The two practical sources are electrically equivalent, then, if

$$R_s = R_p \qquad [16]$$

and

$$v_s = R_p i_s = R_s i_s \qquad [17]$$

where we now let R_s represent the internal resistance of either practical source, which is the conventional notation.

As an illustration of the use of these ideas, consider the practical current source shown in Fig. 5.16*a*. Since its internal resistance is 2 Ω, the internal resistance of the equivalent practical voltage source is also 2 Ω; the voltage of the ideal voltage source contained within the practical voltage source is (2)(3) = 6 V. The equivalent practical voltage source is shown in Fig. 5.16*b*.

To check the equivalence, let us visualize a 4 Ω resistor connected to each source. In both cases a current of 1 A, a voltage of 4 V, and a power of 4 W are associated with the 4 Ω load. However, we should note very carefully that the ideal current source is delivering a total power of 12 W, while the ideal voltage source is delivering only 6 W. Furthermore, the internal resistance of the practical current source is absorbing 8 W, whereas the internal resistance of the practical voltage source is absorbing only 2 W. Thus we see that the two practical sources are equivalent only with respect to what transpires at the load terminals; they are *not* equivalent internally!

EXAMPLE 5.4

Compute the current through the 4.7 kΩ resistor in Fig. 5.17*a* after transforming the 9 mA source into an equivalent voltage source.

The equivalent source consists of an independent voltage source of (9 mA) × (5 kΩ) = 45 V in series with a 5 kΩ resistor, as depicted in Fig. 5.17*b*.

A simple KVL equation around the loop yields:

$$-45 + 5000I + 4700I + 3000I + 3 = 0$$

which can be easily solved to find that the current $I = 3.307$ mA.

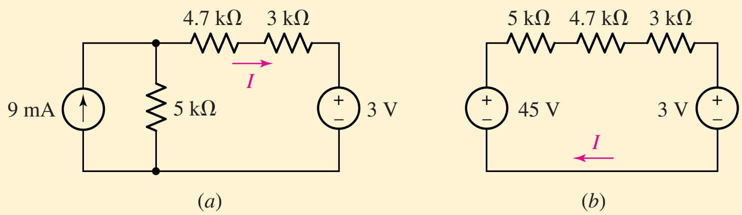


FIGURE 5.17 (*a*) A circuit with both a voltage source and a current source. (*b*) The circuit after the 9 mA source is transformed into an equivalent voltage source.

PRACTICE

5.3 For the circuit of Fig. 5.18, compute the current I_X through the 47 kΩ resistor after performing a source transformation on the voltage source.

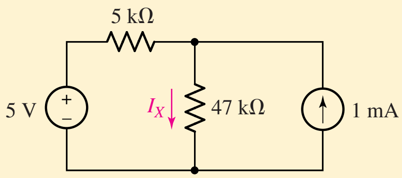


FIGURE 5.18

Ans: 192 μA.

EXAMPLE 5.5

Calculate the current through the 2 Ω resistor in Fig. 5.19*a* on the next page by making use of source transformations to first simplify the circuit.

We begin by transforming each current source into a voltage source (Fig. 5.19*b*), the strategy being to convert the circuit into a simple loop.

We must be careful to retain the 2 Ω resistor for two reasons: first, the dependent source controlling variable appears across it, and second, we desire the current flowing through it. However, we can combine the 17 Ω and 9 Ω resistors, since they appear in series. We also see that the 3 Ω and 4 Ω resistors may be combined into a single 7 Ω resistor, which can then be used to transform the 15 V source into a 15/7 A source as in Fig. 5.19*c*.

As a final simplification, we note that the two 7 Ω resistors can be combined into a single 3.5 Ω resistor, which may be used to transform the 15/7 A current source into a 7.5 V voltage source. The result is a simple loop circuit, shown in Fig. 5.19*d*.

The current I can now be found using KVL:

$$-7.5 + 3.5I - 51V_x + 28I + 9 = 0$$

where

$$V_x = 2I$$

Thus,

$$I = 21.28 \text{ mA}$$

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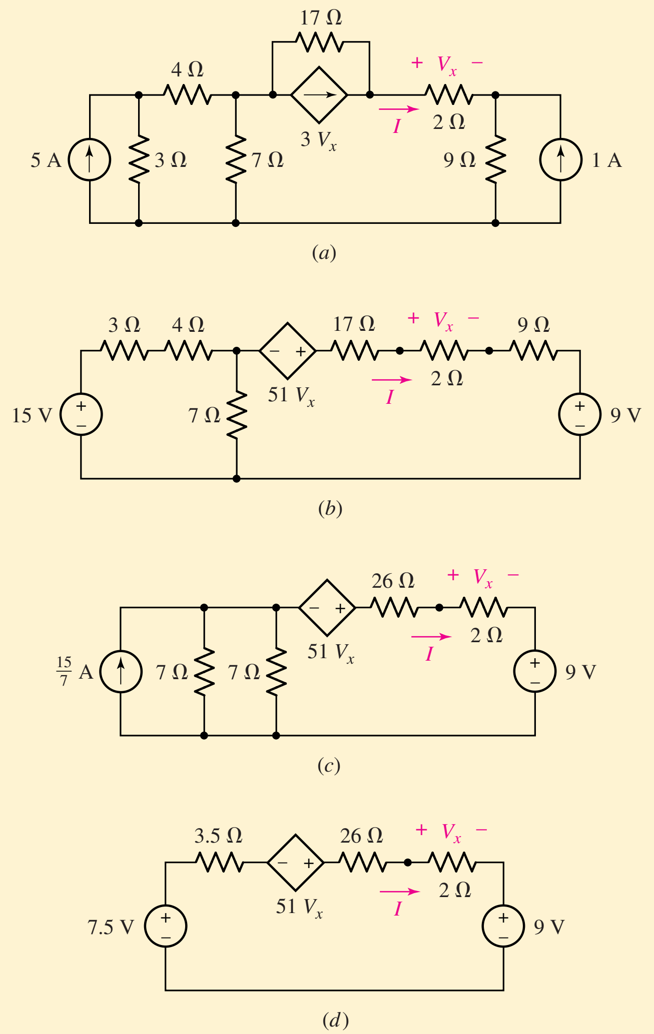
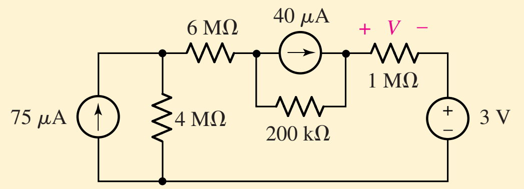


■ **FIGURE 5.19** (*a*) A circuit with two independent current sources and one dependent source. (*b*) The circuit after each source is transformed into a voltage source. (*c*) The circuit after further combinations. (*d*) The final circuit.

PRACTICE

5.4 For the circuit of Fig. 5.20, compute the voltage V across the 1 MΩ resistor using repeated source transformations.



■ **FIGURE 5.20**

Ans: 27.23 V.

Several Key Points

We conclude our discussion of practical sources and source transformations with a few specialized observations. First, when we transform a voltage source, we must be sure that the source is in fact *in series* with the resistor under consideration. For example, in the circuit shown in Fig. 5.21, it is perfectly valid to perform a source transformation on the voltage source using the 10 Ω resistor, as they are in series. However, it would be incorrect to attempt a source transformation using the 60 V source and the 30 Ω resistor—a very common type of error.

In a similar fashion, when we transform a current source and resistor combination, we must be sure that they are in fact *in parallel*. Consider the current source shown in Fig. 5.22*a*. We may perform a source transformation including the 3 Ω resistor, as they are in parallel, but after the transformation there may be some ambiguity as to where to place the resistor. In such circumstances, it is helpful to first redraw the components to be transformed as in Fig. 5.22*b*. Then, the transformation to a voltage source in series with a resistor may be drawn correctly as shown in Fig. 5.22*c*; the resistor may in fact be drawn above or below the voltage source.

It is also worthwhile to consider the unusual case of a current source in series with a resistor and its dual, the case of a voltage source in parallel

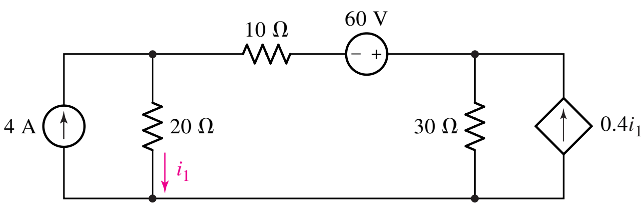


FIGURE 5.21 An example circuit to illustrate how to determine if a source transformation can be performed.

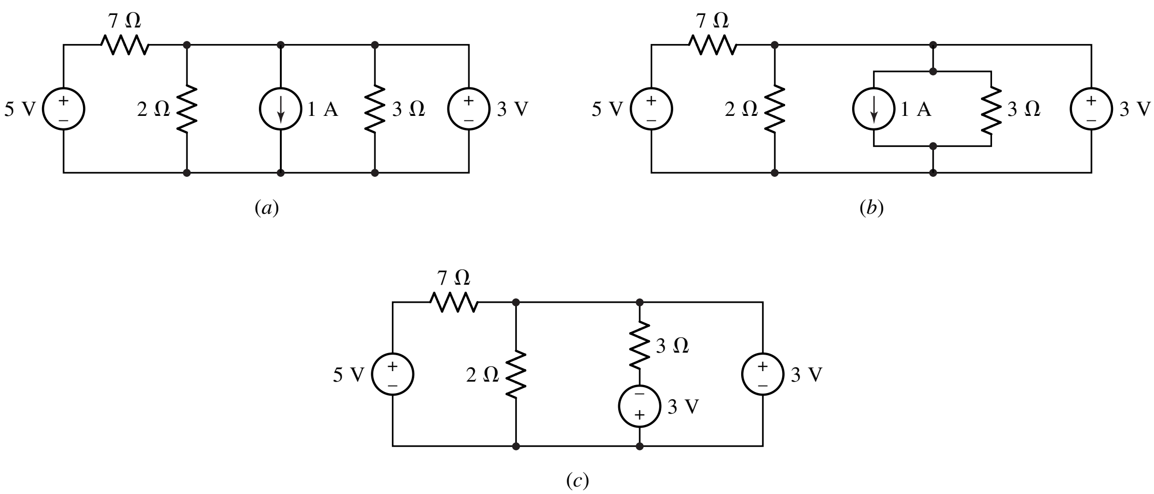
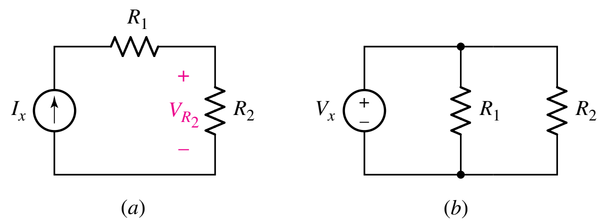


FIGURE 5.22 (*a*) A circuit with a current source to be transformed to a voltage source. (*b*) Circuit redrawn so as to avoid errors. (*c*) Transformed source/resistor combination.

with a resistor. Let's start with the simple circuit of Fig. 5.23*a*, where we are interested only in the voltage across the resistor marked R_2. We note that regardless of the value of resistor R_1, $V_{R_2} = I_x R_2$. Although we might be tempted to perform an inappropriate source transformation on such a circuit, in fact *we may simply omit resistor* R_1 (provided that it is of no interest to us itself). A similar situation arises with a voltage source in parallel with a resistor, as depicted in Fig. 5.23*b*. Again, if we are only interested in some quantity regarding resistor R_2, we may find ourselves tempted to perform some strange (and incorrect) source transformation on the voltage source and resistor R_1. In reality, we may omit resistor R_1 from our circuit as far as resistor R_2 is concerned—its presence does not alter either the voltage across, the current through, or the power dissipated by resistor R_2.



■ **FIGURE 5.23** (*a*) Circuit with a resistor R_1 in series with a current source. (*b*) A voltage source in parallel with two resistors.

Summary of Source Transformation

1. **A common goal in source transformation is to end up with either all current sources or all voltage sources in the circuit.** This is especially true if it makes nodal or mesh analysis easier.
2. **Repeated source transformations can be used to simplify a circuit by allowing resistors and sources to eventually be combined.**
3. **The resistor value does not change during a source transformation, but it is not the same resistor.** This means that currents or voltages associated with the original resistor are irretrievably lost when we perform a source transformation.
4. **If the voltage or current associated with a particular resistor is used as a controlling variable for a dependent source, it should not be included in any source transformation.** The original resistor must be retained in the final circuit, untouched.
5. **If the voltage or current associated with a particular element is of interest, that element should not be included in any source transformation.** The original element must be retained in the final circuit, untouched.
6. **In a source transformation, the head of the current source arrow corresponds to the "+" terminal of the voltage source.**
7. **A source transformation on a current source and resistor requires that the two elements be in parallel.**
8. **A source transformation on a voltage source and resistor requires that the two elements be in series.**

5.3 THÉVENIN AND NORTON EQUIVALENT CIRCUITS

Now that we have been introduced to source transformations and the superposition principle, it is possible to develop two more techniques that will greatly simplify the analysis of many linear circuits. The first of these theorems is named after M. L. Thévenin, a French engineer working in telegraphy who published the theorem in 1883; the second may be considered a corollary of the first and is credited to E. L. Norton, a scientist with the Bell Telephone Laboratories.

Let us suppose that we need to make only a partial analysis of a circuit. For example, perhaps we need to determine the current, voltage, and power delivered to a single "load" resistor by the remainder of the circuit, which may consist of a sizable number of sources and resistors (Fig. 5.24*a*). Or, perhaps we wish to find the response for different values of the load resistance. Thévenin's theorem tells us that it is possible to replace everything except the load resistor with an independent voltage source in series with a resistor (Fig. 5.24*b*); the response measured *at the load resistor* will be unchanged. Using Norton's theorem, we obtain an equivalent composed of an independent current source in parallel with a resistor (Fig. 5.24*c*).

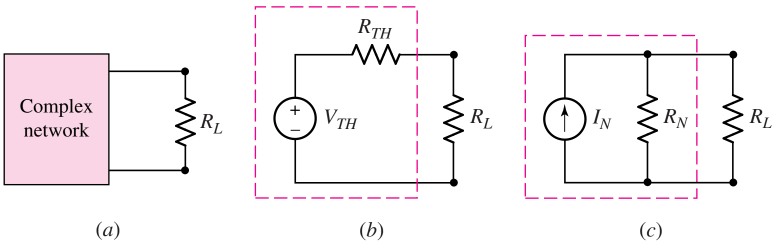


FIGURE 5.24 (*a*) A complex network including a load resistor R_L. (*b*) A Thévenin equivalent network connected to the load resistor R_L. (*c*) A Norton equivalent network connected to the load resistor R_L.

It should thus be apparent that one of the main uses of Thévenin's and Norton's theorems is the replacement of a large part of a circuit, often a complicated and uninteresting part, with a very simple equivalent. The new, simpler circuit enables us to make rapid calculations of the voltage, current, and power which the original circuit is able to deliver to a load. It also helps us to choose the best value of this load resistance. In a transistor power amplifier, for example, the Thévenin or Norton equivalent enables us to determine the maximum power that can be taken from the amplifier and delivered to the speakers.

EXAMPLE 5.6

Consider the circuit shown in Fig. 5.25*a* on the next page. Determine the Thévenin equivalent of network A, and compute the power delivered to the load resistor R_L.

The dashed regions separate the circuit into networks A and B; our main interest is in network B, which consists only of the load resistor R_L. Network A may be simplified by making repeated source transformations.

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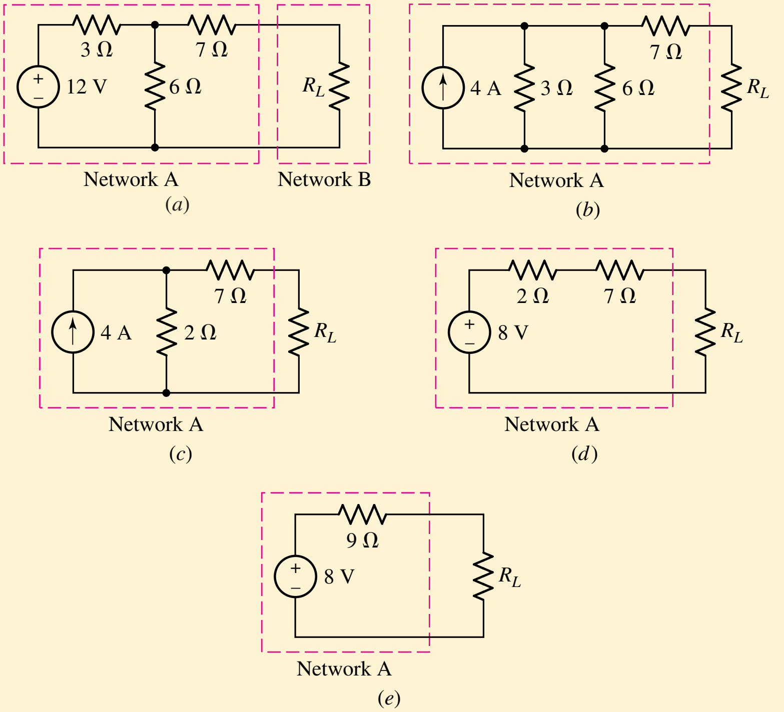

FIGURE 5.25 (*a*) A circuit separated into two networks. (*b*)–(*d*) Intermediate steps to simplifying network *A*. (*e*) The Thévenin equivalent circuit.

We first treat the 12 V source and the 3 Ω resistor as a practical voltage source and replace it with a practical current source consisting of a 4 A source in parallel with 3 Ω (Fig. 5.25*b*). The parallel resistances are then combined into 2 Ω (Fig. 5.25*c*), and the practical current source that results is transformed back into a practical voltage source (Fig. 5.25*d*). The final result is shown in Fig. 5.25*e*.

From the viewpoint of the load resistor R_L, this network *A* (the Thévenin equivalent) is equivalent to the original network *A*; from our viewpoint, the circuit is much simpler, and we can now easily compute the power delivered to the load:

$$P_L = \left(\frac{8}{9 + R_L}\right)^2 R_L$$

Furthermore, we can see from the equivalent circuit that the maximum voltage that can be obtained across R_L is 8 V and corresponds to $R_L = \infty$. A quick transformation of network *A* to a practical current source (the Norton equivalent) indicates that the maximum current that may be delivered to the load is 8/9 A, which occurs when $R_L = 0$. Neither of these facts is readily apparent from the original circuit.

PRACTICE

5.5 Using repeated source transformations, determine the Norton equivalent of the highlighted network in the circuit of Fig. 5.26.

Ans: 1 A, 5 Ω.

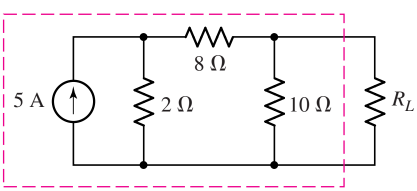

FIGURE 5.26

Thévenin's Theorem

Using the technique of source transformation to find a Thévenin or Norton equivalent network worked well enough in Example 5.6, but it can rapidly become impractical in situations where dependent sources are present or the circuit is composed of a large number of elements. An alternative is to employ Thévenin's theorem (or Norton's theorem) instead. We will state the theorem as a somewhat formal procedure and then proceed to consider various ways to make the approach more practical depending on the situation we face.

A Statement of Thévenin's Theorem[3]

1. **Given any linear circuit, rearrange it in the form of two networks, *A* and *B*, connected by two wires.** *A* is the network to be simplified; *B* will be left untouched.
2. **Disconnect network *B*.** Define a voltage v_{oc} as the voltage now appearing across the terminals of network *A*.
3. **Turn off or "zero out" every independent source in network *A* to form an inactive network.** Leave dependent sources unchanged.
4. **Connect an independent voltage source with value v_{oc} in series with the inactive network.** Do not complete the circuit; leave the two terminals disconnected.
5. **Connect network *B* to the terminals of the new network *A*.** All currents and voltages in *B* will remain unchanged.

Note that if either network contains a dependent source, *its control variable must be in the same network*.

CAUTION

Let us see if we can apply Thévenin's theorem successfully to the circuit we considered in Fig. 5.25. We have already found the Thévenin equivalent of the circuit to the left of R_L in Example 5.6, but we want to see if there is an easier way to obtain the same result.

EXAMPLE 5.7

Use Thévenin's theorem to determine the Thévenin equivalent for that part of the circuit in Fig. 5.25*a* to the left of R_L.

We begin by disconnecting R_L, and note that no current flows through the 7 Ω resistor in the resulting partial circuit shown in Fig. 5.27*a* on the next page. Thus, V_{oc} appears across the 6 Ω resistor (with no current through the 7 Ω resistor there is no voltage drop across it), and voltage division enables us to determine that

$$V_{oc} = 12\left(\frac{6}{3+6}\right) = 8 \text{ V}$$

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(3) A proof of Thévenin's theorem in the form in which we have stated it is rather lengthy, and therefore it has been placed in App. 3, where the curious may peruse it.

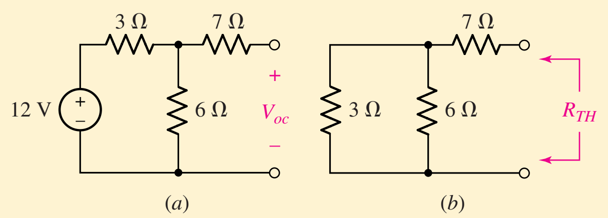


FIGURE 5.27 (*a*) The circuit of Fig. 5.25*a* with network *B* (the resistor R_L) disconnected and the voltage across the connecting terminals labeled as V_{oc}. (*b*) The independent source in Fig. 5.25*a* has been killed, and we look into the terminals where network *B* was connected to determine the effective resistance of network *A*.

Killing network *A* (i.e., replacing the 12 V source with a short circuit), we see looking back into the dead network a 7 Ω resistor connected in series with the parallel combination of 6 Ω and 3 Ω (Fig. 5.27*b*).

Thus, the dead network can be represented here by a 9 Ω resistor, referred to as the ***Thévenin equivalent resistance*** of network *A*. The Thévenin equivalent then is V_{oc} in series with a 9 Ω resistor, which agrees with our previous result.

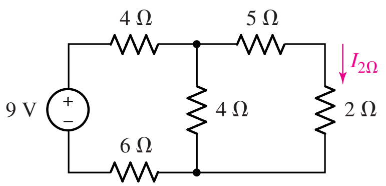


FIGURE 5.28

PRACTICE

5.6 Use Thévenin's theorem to find the current through the 2 Ω resistor in the circuit of Fig. 5.28. (Hint: Designate the 2 Ω resistor as network *B*.)

Ans: $V_{TH} = 2.571$ V, $R_{TH} = 7.857$ Ω, $I_{2\Omega} = 260.8$ mA.

A Few Key Points

The equivalent circuit we have learned how to obtain is completely independent of network *B*, because we have been instructed first to remove network *B* and then measure the open-circuit voltage produced by network *A*, an operation that certainly does not depend on network *B* in any way. The *B* network is mentioned in the statement of the theorem only to indicate that an equivalent for *A* may be obtained no matter what arrangement of elements is connected to the *A* network; the *B* network represents this general network.

There are several points about the theorem which deserve emphasis.

- The only restriction that we must impose on *A* or *B* is that all *dependent* sources in *A* have their control variables in *A*, and similarly for *B*.
- No restrictions were imposed on the complexity of *A* or *B*; either one may contain any combination of independent voltage or current sources, linear dependent voltage or current sources, resistors, or any other circuit elements which are linear.
- The dead network *A* can be represented by a single equivalent resistance R_{TH}, which we will call the Thévenin equivalent resistance.

This holds true whether or not dependent sources exist in the dead *A* network, an idea we will explore shortly.

- A Thévenin equivalent consists of two components: a voltage source in series with a resistance. Either may be zero, although this is not usually the case.

Norton's Theorem

Norton's theorem bears a close resemblance to Thévenin's theorem and may be stated as follows:

A Statement of Norton's Theorem

1. **Given any linear circuit, rearrange it in the form of two networks, *A* and *B*, connected by two wires.** *A* is the network to be simplified; *B* will be left untouched. As before, if either network contains a dependent source, *its controlling variable must be in the same network.*
2. **Disconnect network *B*, and short the terminals of *A*.** Define a current i_{sc} as the current now flowing through the shorted terminals of network *A*.
3. **Turn off or "zero out" every independent source in network *A* to form an inactive network.** Leave dependent sources unchanged.
4. **Connect an independent current source with value i_{sc} in parallel with the inactive network.** Do not complete the circuit; leave the two terminals disconnected.
5. **Connect network *B* to the terminals of the new network *A*.** All currents and voltages in *B* will remain unchanged.

The Norton equivalent of a linear network is the Norton current source i_{sc} in parallel with the Thévenin resistance R_{TH}. Thus, we see that in fact it is possible to obtain the Norton equivalent of a network by performing a source transformation on the Thévenin equivalent. This results in a direct relationship between v_{oc}, i_{sc}, and R_{TH}:

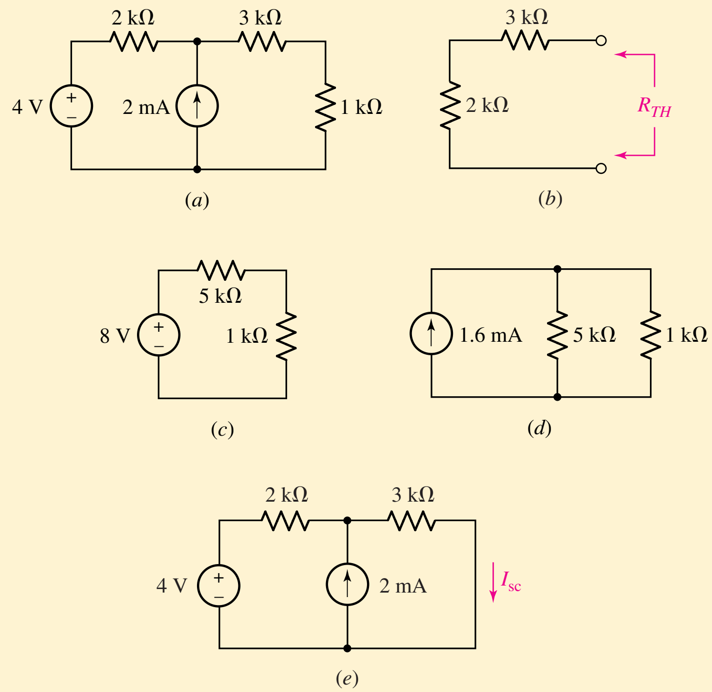
$$\boxed{v_{oc} = R_{TH} i_{sc}} \qquad [18]$$

In circuits containing dependent sources, we will often find it more convenient to determine either the Thévenin or Norton equivalent by finding both the open-circuit voltage and the short-circuit current and then determining the value of R_{TH} as their quotient. It is therefore advisable to become adept at finding both open-circuit voltages and short-circuit currents, even in the simple problems that follow. If the Thévenin and Norton equivalents are determined independently, Eq. [18] can serve as a useful check.

Let us consider three different examples of the determination of a Thévenin or Norton equivalent circuit.

EXAMPLE 5.8

Find the Thévenin and Norton equivalent circuits for the network faced by the 1 kΩ resistor in Fig. 5.29*a*.



■ **FIGURE 5.29** (*a*) A given circuit in which the 1 kΩ resistor is identified as network *B*. (*b*) Network *A* with all independent sources killed. (*c*) The Thévenin equivalent is shown for network *A*. (*d*) The Norton equivalent is shown for network *A*. (*e*) Circuit for determining I_{sc}.

From the way the problem statement is worded, we know that network *B* is the 1 kΩ resistor, and network *A* is the remainder of the circuit. The circuit contains no dependent sources, and the easiest way to find the Thévenin equivalent is to determine R_{TH} for the dead network directly, followed by a calculation of either V_{oc} or I_{sc}.

We first determine the open-circuit voltage; in this case it is easily found by superposition. With only the 4 V source operating, the open-circuit voltage is 4 V; when only the 2 mA source is on, the open-circuit voltage is 2 mA × 2 kΩ $= 4$ V (no current flows through the 3 kΩ resistor with the 1 kΩ resistor disconnected). With both independent sources on, we see that $V_{oc} = 4 + 4 = 8$ V.

We next kill both independent sources to determine the form of the dead *A* network. With the 4 V source short-circuited and the 2 mA source open-circuited as in Fig. 5.29*b*, the result is the series combination of a 2 kΩ and a 3 kΩ resistor, or the equivalent, a 5 kΩ resistor.

This determines the Thévenin equivalent, shown in Fig. 5.29*c*, and from it the Norton equivalent of Fig. 5.29*d* can be drawn quickly. As a check, let us determine I_{sc} for the given circuit (Fig. 5.29*e*). We use superposition and a little current division:

$$I_{sc} = I_{sc}|_{4\ V} + I_{sc}|_{2\ mA} = \frac{4}{2+3} + (2)\frac{2}{2+3}$$
$$= 0.8 + 0.8 = 1.6\ \text{mA}$$

which completes the check.[4]

PRACTICE

5.7 Determine the Thévenin and Norton equivalents of the circuit of Fig. 5.30.

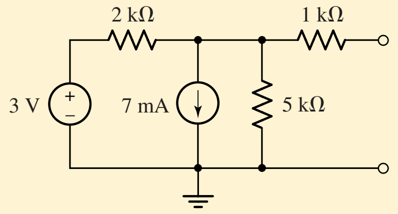


FIGURE 5.30

Ans: –7.857 V, –3.235 mA, 2.429 kΩ.

When Dependent Sources Are Present

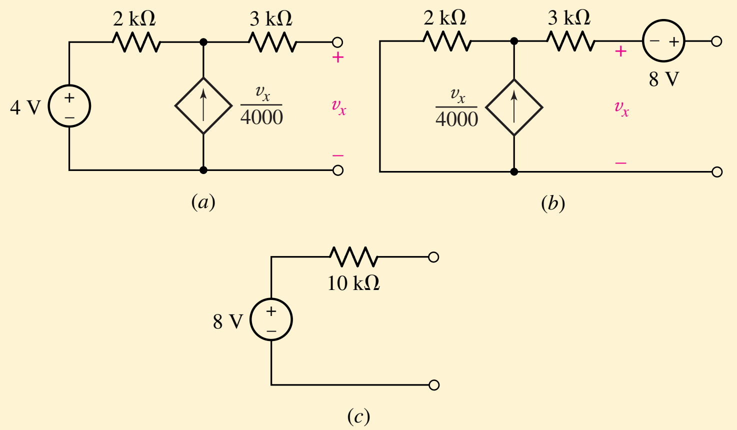
Technically speaking, there does not always have to be a "network *B*" for us to invoke either Thévenin's theorem or Norton's theorem; we could instead be asked to find the equivalent of a network with two terminals not yet connected to another network. If there *is* a network *B* that we do not want to involve in the simplification procedure, however, we must use a little caution if it contains dependent sources. In such situations, the controlling variable and the associated element(s) must be included in network *B* and excluded from network *A*. Otherwise, there will be no way to analyze the final circuit because the controlling quantity will be lost.

If network *A* contains a dependent source, then again we must ensure that the controlling variable and its associated element(s) cannot be in network *B*. Up to now, we have only considered circuits with resistors and independent sources. Although technically speaking it is correct to leave a dependent source in the "dead" or "inactive" network when creating a Thévenin or Norton equivalent, in practice this does not result in any kind of simplification. What we really want is an independent voltage source in series with a single resistor, or an independent current source in parallel with a single resistor—in other words, a two-component equivalent. In the following examples, we consider various means of reducing networks with dependent sources and resistors into a single resistance.

(4) Note: If we use resistance in kΩ throughout our equation and voltage is expressed in volts, then the current will always automatically be in mA.

EXAMPLE 5.9

Determine the Thévenin equivalent of the circuit in Fig. 5.31*a*.



■ **FIGURE 5.31** (*a*) A given network whose Thévenin equivalent is desired. (*b*) A possible, but rather useless, form of the Thévenin equivalent. (*c*) The best form of the Thévenin equivalent for this linear resistive network.

To find V_{oc} we note that $v_x = V_{oc}$ and that the dependent source current must pass through the 2 kΩ resistor, since no current can flow through the 3 kΩ resistor. Using KVL around the outer loop:

$$-4 + 2 \times 10^3 \left(-\frac{v_x}{4000}\right) + 3 \times 10^3(0) + v_x = 0$$

and

$$v_x = 8 \text{ V} = V_{oc}$$

By Thévenin's theorem, then, the equivalent circuit could be formed with the dead *A* network in series with an 8 V source, as shown in Fig. 5.31*b*. This is correct, but not very simple and not very helpful; in the case of linear resistive networks, we should certainly show a much simpler equivalent for the inactive *A* network, namely, R_{TH}.

The presence of the dependent source prevents us from determining R_{TH} directly for the inactive network through resistance combination; we therefore seek I_{sc}. Upon short-circuiting the output terminals in Fig. 5.31*a*, it is apparent that $V_x = 0$ and the dependent current source is dead. Hence, $I_{sc} = 4/(5 \times 10^3) = 0.8$ mA. Thus,

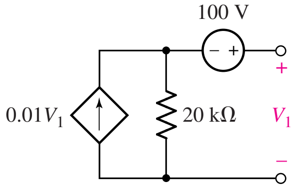
$$R_{TH} = \frac{V_{oc}}{I_{sc}} = \frac{8}{(0.8 \times 10^{-3})} = 10 \text{ k}\Omega$$

and the acceptable Thévenin equivalent of Fig. 5.31*c* is obtained.

PRACTICE

5.8 Find the Thévenin equivalent for the network of Fig. 5.32. (Hint: A quick source transformation on the dependent source might help.)

Ans: −502.5 mV, −100.5 Ω.



■ **FIGURE 5.32**

As our final example, let us consider a network having a dependent source but no independent source.

EXAMPLE 5.10

Find the Thévenin equivalent of the circuit shown in Fig. 5.33*a*.

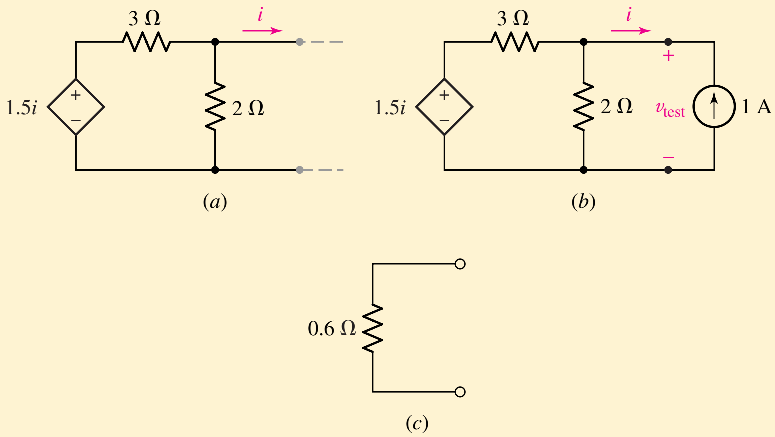


FIGURE 5.33 (*a*) A network with no independent sources. (*b*) A hypothetical measurement to obtain R_{TH}. (*c*) The Thévenin equivalent to the original circuit.

Since the rightmost terminals are already open-circuited, $i = 0$. Consequently, the dependent source is dead, so $v_{oc} = 0$.

We next seek the value of R_{TH} represented by this two-terminal network. However, we cannot find v_{oc} and i_{sc} and take their quotient, for there is no independent source in the network and both v_{oc} and i_{sc} are zero. Let us, therefore, be a little tricky.

We apply a 1 A source externally, measure the voltage v_{test} that results, and then set $R_{TH} = v_{test}/1$. Referring to Fig. 5.33*b*, we see that $i = -1$ A. Applying nodal analysis,

$$\frac{v_{test} - 1.5(-1)}{3} + \frac{v_{test}}{2} = 1$$

so that

$$v_{test} = 0.6 \text{ V}$$

and thus

$$R_{TH} = 0.6\ \Omega$$

The Thévenin equivalent is shown in Fig. 5.33*c*.

A Quick Recap of Procedures

We have now looked at three examples in which we determined a Thévenin or Norton equivalent circuit. The first example (Fig. 5.29) contained only independent sources and resistors, and several different methods could have been applied to it. One would involve calculating R_{TH} for the dead network and then V_{oc} for the live network. We could also have found R_{TH} and I_{sc}, or V_{oc} and I_{sc}.

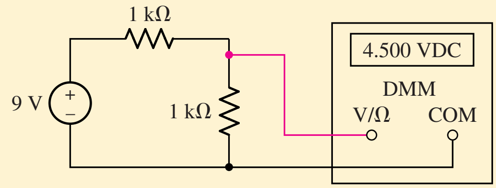
PRACTICAL APPLICATION

The Digital Multimeter

One of the most common pieces of electrical test equipment is the DMM, or digital multimeter (Fig. 5.34), which is designed to measure voltage, current, and resistance values.

■ **FIGURE 5.34** A handheld digital multimeter.

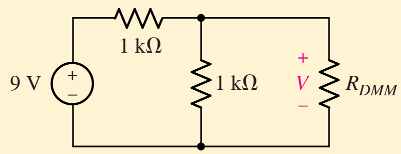
In a voltage measurement, two leads from the DMM are connected across the appropriate circuit element, as depicted in Fig. 5.35. The positive reference terminal of the meter is typically marked "V/Ω," and the negative



■ **FIGURE 5.35** A DMM connected to measure voltage.

reference terminal—often referred to as the *common terminal*—is typically designated by "COM." The typical convention is to use a red-colored lead for the positive reference terminal and a black lead for the common terminal.

From our discussion of Thévenin and Norton equivalents, it may now be apparent that the DMM has its own Thévenin equivalent resistance. This Thévenin equivalent resistance will appear in parallel with our circuit, and its value can affect the measurement (Fig. 5.36). The DMM does not supply power to the circuit to measure voltage, so its Thévenin equivalent consists of only a resistance, which we will name R_{DMM}.



■ **FIGURE 5.36** DMM in Fig. 5.35 shown as its Thévenin equivalent resistance, R_{DMM}.

The input resistance of a good DMM is typically 10 MΩ or more. The measured voltage V thus appears

In the second example (Fig. 5.31), both independent and dependent sources were present, and the method we used required us to find V_{oc} and I_{sc}. We could not easily find R_{TH} for the dead network because the dependent source could not be made inactive.

CAUTION

The last example did not contain any independent sources, and therefore the Thévenin and Norton equivalents do not contain an independent source. We found R_{TH} by applying 1 A and finding $v_{\text{test}} = 1 \times R_{TH}$. We could also apply 1 V and determine $i = 1/R_{TH}$. These two related techniques can be applied to any circuit with dependent sources, *as long as all independent sources are set to zero first*.

Two other methods have a certain appeal because they can be used for any of the three types of networks considered. In the first, simply replace network B with a voltage source v_s, define the current leaving its positive terminal as i, then analyze network A to obtain i, and put the equation in the form $v_s = ai + b$. Then, $a = R_{TH}$ and $b = v_{\text{oc}}$.

across 1 kΩ‖10 MΩ = 999.9 Ω. Using voltage division, we find that $V = 4.4998$ volts, slightly less than the expected value of 4.5 volts. Thus, the finite input resistance of the voltmeter introduces a small error in the measured value.

To measure currents, the DMM must be placed in series with a circuit element, generally requiring that we cut a wire (Fig. 5.37). One DMM lead is connected to the common terminal of the meter, and the other lead is placed in a connector usually marked "A" to signify current measurement. Again, the DMM does not supply power to the circuit in this type of measurement.

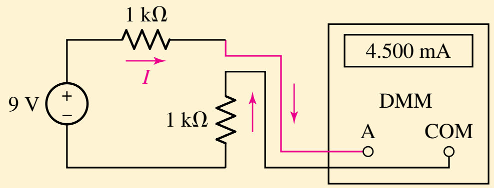


FIGURE 5.37 A DMM connected to measure current.

We see from this figure that the Thévenin equivalent resistance (R_{DMM}) of the DMM is in series with our circuit, so its value can affect the measurement. Writing a simple KVL equation around the loop,

$$-9 + 1000I + R_{\text{DMM}}I + 1000I = 0$$

Note that since we have reconfigured the meter to perform a current measurement, the Thévenin equivalent resistance is not the same as when the meter is configured to measure voltages. In fact, we would ideally like R_{DMM} to be 0 Ω for current measurements, and ∞ for voltage measurements. If R_{DMM} is now 0.1 Ω, we see that the measured current I is 4.4998 mA, which is only slightly different from the expected value of 4.5 mA. Depending on the number of digits that can be displayed by the meter, we may not even notice the effect of nonzero DMM resistance on our measurement.

The same meter can be used to determine resistance, provided no independent sources are active during the measurement. Internally, a known current is passed through the resistor being measured, and the voltmeter circuitry is used to measure the resulting voltage. Replacing the DMM with its Norton equivalent (which now includes an active independent current source to generate the predetermined current), we see that R_{DMM} appears in parallel with our unknown resistor R (Fig. 5.38).

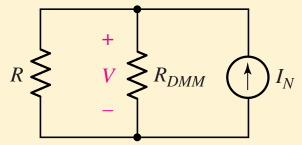


FIGURE 5.38 DMM in resistance measurement configuration replaced by its Norton equivalent, showing R_{DMM} in parallel with the unknown resistor R to be measured.

As a result, the DMM actually measures $R \| R_{\text{DMM}}$. If $R_{\text{DMM}} = 10$ MΩ and $R = 10$ Ω, $R_{\text{measured}} = 9.99999$ Ω, which is more than accurate enough for most purposes. However, if $R = 10$ MΩ, $R_{\text{measured}} = 5$ MΩ. The input resistance of a DMM therefore places a practical upper limit on the values of resistance that can be measured, and special techniques must be used to measure larger resistances. We should note that if a digital multimeter is *programmed* with knowledge of R_{DMM}, it is possible to compensate and allow measurement of larger resistances.

We could also apply a current source i_s, let its voltage be v, and then determine $i_s = cv - d$, where $c = 1/R_{TH}$ and $d = i_{\text{sc}}$ (the minus sign arises from assuming both current source arrows are directed into the same node). Both of these last two procedures are universally applicable, but some other method can usually be found that is easier and more rapid.

Although we are devoting our attention almost entirely to the analysis of linear circuits, it is good to know that Thévenin's and Norton's theorems are both valid if network B is nonlinear; only network A must be linear.

PRACTICE

5.9 Find the Thévenin equivalent for the network of Fig. 5.39. (Hint: Try a 1 V test source.)

Ans: $I_{\text{test}} = 50$ mA so $R_{TH} = 20$ Ω.

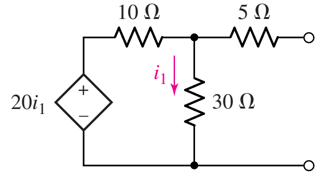
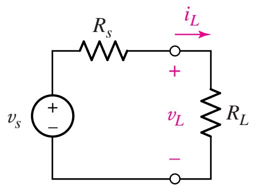


FIGURE 5.39 See Practice Problem 5.9.



■ **FIGURE 5.40** A practical voltage source connected to a load resistor R_L.

5.4 MAXIMUM POWER TRANSFER

A very useful power theorem may be developed with reference to a practical voltage or current source. For the practical voltage source (Fig. 5.40), the power delivered to the load R_L is

$$p_L = i_L^2 R_L = \frac{v_s^2 R_L}{(R_s + R_L)^2} \qquad [19]$$

To find the value of R_L that absorbs a maximum power from the given practical source, we differentiate with respect to R_L:

$$\frac{d\,p_L}{d\,R_L} = \frac{(R_s + R_L)^2 v_s^2 - v_s^2 R_L (2)(R_s + R_L)}{(R_s + R_L)^4}$$

and equate the derivative to zero, obtaining

$$2R_L(R_s + R_L) = (R_s + R_L)^2$$

or

$$R_s = R_L$$

Since the values $R_L = 0$ and $R_L = \infty$ both give a minimum ($p_L = 0$), and since we have already developed the equivalence between practical voltage and current sources, we have therefore proved the following ***maximum power transfer theorem:***

> An independent voltage source in series with a resistance R_s, or an independent current source in parallel with a resistance R_s, delivers a maximum power to that load resistance R_L for which $R_L = R_s$.

It may have occurred to the reader that an alternative way to view the maximum power theorem is possible in terms of the Thévenin equivalent resistance of a network:

> A network delivers the maximum power to a load resistance R_L when R_L is equal to the Thévenin equivalent resistance of the network.

Thus, the maximum power transfer theorem tells us that a 2 Ω resistor draws the greatest power (4.5 W) from either practical source of Fig. 5.16, whereas a resistance of 0.01 Ω receives the maximum power (3.6 kW) in Fig. 5.11.

There is a distinct difference between *drawing* maximum power from a *source* and *delivering* maximum power to a *load*. If the load is sized such that its Thévenin resistance is equal to the Thévenin resistance of the network to which it is connected, it will receive maximum power from that network. *Any change to the load resistance will reduce the power delivered to the load*. However, consider just the Thévenin equivalent of the network itself. We draw the maximum possible power from the voltage source by drawing the maximum possible current—which is achieved by shorting the network terminals! However, in this extreme example *we deliver zero power* to the "load"—a short circuit in this case—as $p = i^2 R$, and we just set $R = 0$ by shorting the network terminals.

A minor amount of algebra applied to Eq. [19] coupled with the maximum power transfer requirement that $R_L = R_s = R_{TH}$ will provide

$$p_{\max}\big|_{\text{delivered to load}} = \frac{v_s^2}{4R_s} = \frac{v_{TH}^2}{4R_{TH}}$$

where v_{TH} and R_{TH} recognize that the practical voltage source of Fig. 5.40 can also be viewed as a Thévenin equivalent of some specific source.

It is also not uncommon for the maximum power theorem to be misinterpreted. It is designed to help us select an optimum load in order to maximize power absorption. If the load resistance is already specified, however, the maximum power theorem is of no assistance. If for some reason we can affect the size of the Thévenin equivalent resistance of the network connected to our load, setting it equal to the load does not guarantee maximum power transfer to our predetermined load. A quick consideration of the power lost in the Thévenin resistance will clarify this point.

CAUTION

EXAMPLE 5.11

The circuit shown in Fig. 5.41 is a model for the common-emitter bipolar junction transistor amplifier. Choose a load resistance so that maximum power is transferred to it from the amplifier, and calculate the actual power absorbed.

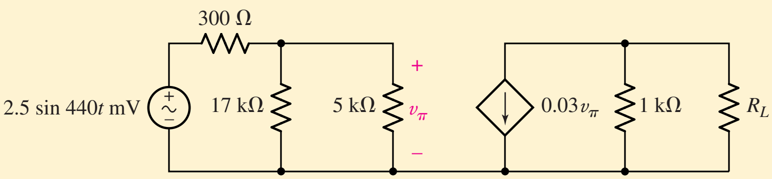


FIGURE 5.41 A small-signal model of the common-emitter amplifier, with the load resistance unspecified.

Since it is the load resistance we are asked to determine, the maximum power theorem applies. The first step is to find the Thévenin equivalent of the rest of the circuit.

We first determine the Thévenin equivalent resistance, which requires that we remove R_L and short-circuit the independent source as in Fig. 5.42*a*.

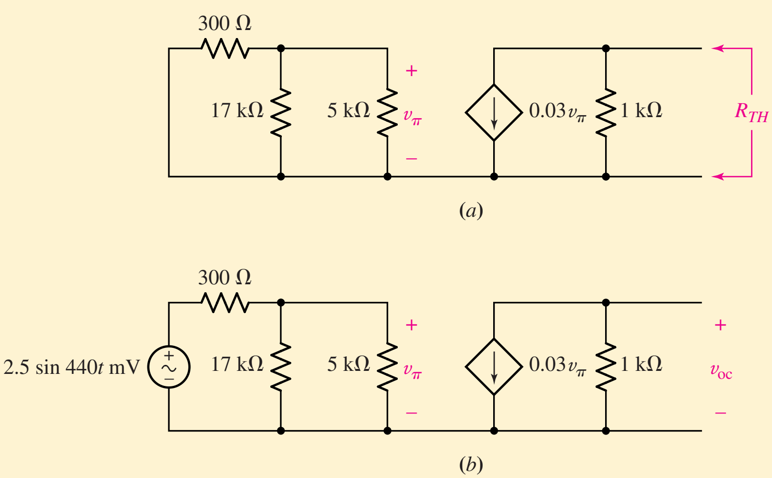


FIGURE 5.42 (*a*) Circuit with R_L removed and independent source short-circuited. (*b*) Circuit for determining v_{TH}.

(*Continued on next page*)

Since $v_\pi = 0$, the dependent current source is an open circuit, so $R_{TH} = 1\ \text{k}\Omega$. This can be verified by connecting an independent 1 A current source across the 1 kΩ resistor; v_π will still be zero, so the dependent source remains inactive and hence contributes nothing to R_{TH}.

In order to obtain maximum power delivered into the load, R_L should be set to $\boxed{R_{TH} = 1\ \text{k}\Omega.}$

To find v_{TH} we consider the circuit shown in Fig. 5.42*b*, which is Fig. 5.41 with R_L removed. We may write

$$v_{\text{oc}} = -0.03v_\pi(1000) = -30v_\pi$$

where the voltage v_π may be found from simple voltage division:

$$v_\pi = (2.5 \times 10^{-3} \sin 440t)\left(\frac{3864}{300 + 3864}\right)$$

so that our Thévenin equivalent is a voltage $-69.6 \sin 440t$ mV in series with 1 kΩ.

The maximum power is given by

$$p_{\max} = \frac{v_{TH}^2}{4R_{TH}} = \boxed{1.211 \sin^2 440t\ \mu\text{W}}$$

PRACTICE

5.10 Consider the circuit of Fig. 5.43.

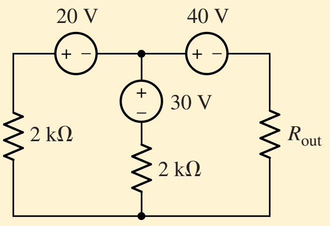


FIGURE 5.43

(*a*) If $R_{\text{out}} = 3\ \text{k}\Omega$, find the power delivered to it.

(*b*) What is the maximum power that can be delivered to any R_{out}?

(*c*) What two different values of R_{out} will have exactly 20 mW delivered to them?

Ans: 230 mW; 306 mW; 59.2 kΩ and 16.88 Ω.

5.5 DELTA-WYE CONVERSION

We saw previously that identifying parallel and series combinations of resistors can often lead to a significant reduction in the complexity of a circuit. In situations where such combinations do not exist, we can often make use of source transformations to enable such simplifications. There is another useful technique, called ***Δ-Y (delta-wye)*** conversion, that arises out of network theory.

Consider the circuits in Fig. 5.44. There are no series or parallel combinations that can be made to further simplify any of the circuits (note that 5.44*a* and 5.44*b* are identical, as are 5.44*c* and 5.44*d*), and without any

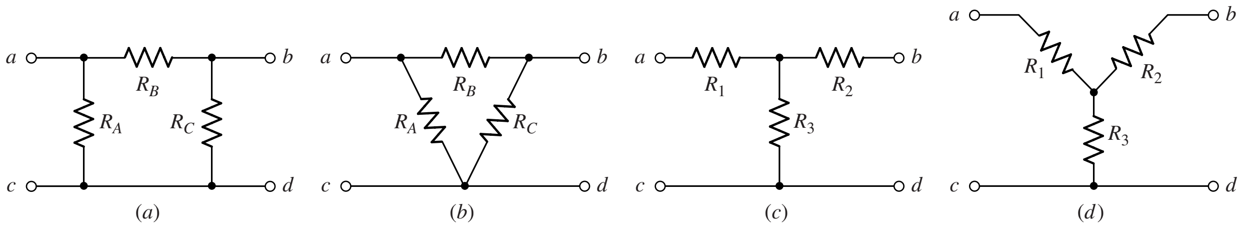


FIGURE 5.44 (*a*) Π network consisting of three resistors and three unique connections. (*b*) Same network drawn as a Δ network. (*c*) A T network consisting of three resistors. (*d*) Same network drawn as a Y network.

sources present, no source transformations can be performed. However, it is possible to convert between these two types of networks.

We first define two voltages v_{ab} and v_{cd}, and three currents i_1, i_2, and i_3 as depicted in Fig. 5.45. If the two networks are equivalent, then the terminal voltages and currents must be equal (there is no current i_2 in the T-connected network). A set of relationships between R_A, R_B, R_C and R_1, R_2, and R_3 can now be defined simply by performing mesh analysis. For example, for the network of Fig. 5.45*a* we may write

$$R_A i_1 - R_A i_2 = v_{ac} \qquad [20]$$

$$-R_A i_1 + (R_A + R_B + R_C) i_2 - R_C i_3 = 0 \qquad [21]$$

$$-R_C i_2 + R_C i_3 = -v_{bc} \qquad [22]$$

and for the network of Fig. 5.45*b* we have

$$(R_1 + R_3) i_1 - R_3 i_3 = v_{ac} \qquad [23]$$

$$-R_3 i_1 + (R_2 + R_3) i_3 = -v_{bc} \qquad [24]$$

We next remove i_2 from Eqs. [20] and [22] using Eq. [21], resulting in

$$\left(R_A - \frac{R_A^2}{R_A + R_B + R_C}\right) i_1 - \frac{R_A R_C}{R_A + R_B + R_C} i_3 = v_{ac} \qquad [25]$$

and

$$-\frac{R_A R_C}{R_A + R_B + R_C} i_1 + \left(R_C - \frac{R_C^2}{R_A + R_B + R_C}\right) i_3 = -v_{bc} \qquad [26]$$

Comparing terms between Eq. [25] and Eq. [23], we see that

$$R_3 = \frac{R_A R_C}{R_A + R_B + R_C}$$

In a similar fashion, we may find expressions for R_1 and R_2 in terms of R_A, R_B, and R_C, as well as expressions for R_A, R_B, and R_C in terms of R_1, R_2, and R_3; we leave the remainder of the derivations as an exercise for the reader. Thus, to convert from a Y network to a Δ network, the new resistor values are calculated using

$$\boxed{\begin{aligned} R_A &= \frac{R_1 R_2 + R_2 R_3 + R_3 R_1}{R_2} \\ R_B &= \frac{R_1 R_2 + R_2 R_3 + R_3 R_1}{R_3} \\ R_C &= \frac{R_1 R_2 + R_2 R_3 + R_3 R_1}{R_1} \end{aligned}}$$

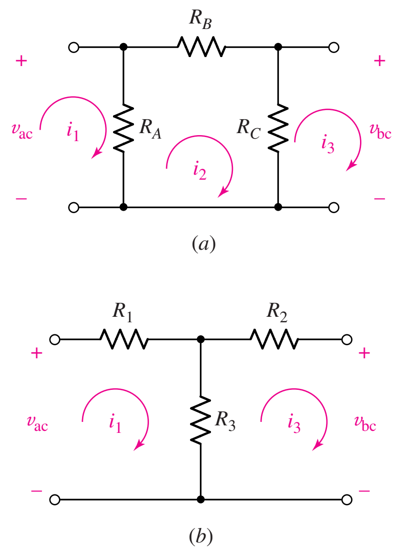


FIGURE 5.45 (*a*) Labeled Π network; (*b*) labeled T network.

and to convert from a Δ network to a Y network,

$$\boxed{\begin{aligned} R_1 &= \frac{R_A R_B}{R_A + R_B + R_C} \\ R_2 &= \frac{R_B R_C}{R_A + R_B + R_C} \\ R_3 &= \frac{R_C R_A}{R_A + R_B + R_C} \end{aligned}}$$

Application of these equations is straightforward, although identifying the actual networks sometimes requires a little concentration.

EXAMPLE 5.12

Use the technique of Δ-Y conversion to find the Thévenin equivalent resistance of the circuit in Fig. 5.46*a*.

We see that the network in Fig. 5.46*a* is composed of two Δ-connected networks that share the 3 Ω resistor. We must be careful at this point not to be too eager, attempting to convert both Δ-connected networks to two Y-connected networks. The reason for this may be more obvious after we convert the top network consisting of the 1, 4, and 3 Ω resistors into a Y-connected network (Fig. 5.46*b*).

Note that in converting the upper network to a Y-connected network, we have removed the 3 Ω resistor. As a result, there is no way to convert the original Δ-connected network consisting of the 2, 5, and 3 Ω resistors into a Y-connected network.

We proceed by combining the $\frac{3}{8}$ Ω and 2 Ω resistors and the $\frac{3}{2}$ Ω and 5 Ω resistors (Fig. 5.46*c*). We now have a $\frac{19}{8}$ Ω resistor in parallel with a $\frac{13}{2}$ Ω resistor, and this parallel combination is in series with the $\frac{1}{2}$ Ω resistor. Thus, we can replace the original network of Fig. 5.46*a* with a single $\frac{159}{71}$ Ω resistor (Fig. 5.46*d*).

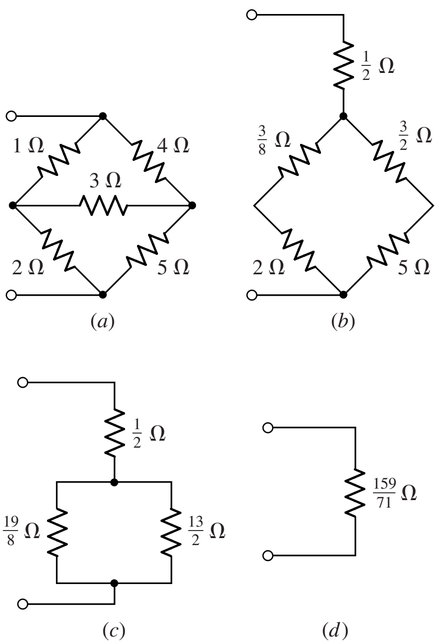


FIGURE 5.46 (*a*) A given resistive network whose input resistance is desired. (*b*) The upper Δ network is replaced by an equivalent Y network. (*c*, *d*) Series and parallel combinations result in a single resistance value.

PRACTICE

5.11 Use the technique of Y-Δ conversion to find the Thévenin equivalent resistance of the circuit of Fig. 5.47.

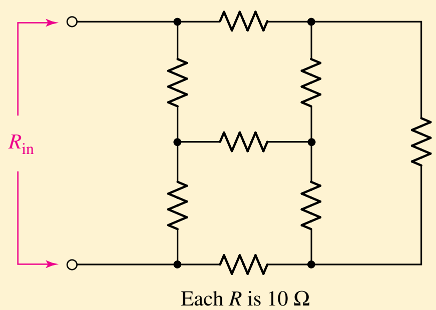


FIGURE 5.47

Ans: 11.43 Ω.

5.6 SELECTING AN APPROACH: A SUMMARY OF VARIOUS TECHNIQUES

In Chap. 3, we were introduced to Kirchhoff's current law (KCL) and Kirchhoff's voltage law (KVL). These two laws apply to any circuit we will ever encounter, provided that we take care to consider the entire system that the circuits represent. The reason for this is that KCL and KVL enforce charge and energy conservation, respectively, which are very fundamental principles. Based on KCL, we developed the very powerful method of nodal analysis. A similar technique based on KVL (unfortunately only applicable to planar circuits) is known as mesh analysis and is also a useful circuit analysis approach.

For the most part, this text is concerned with developing analytical skills that apply to *linear* circuits. If we know a circuit is constructed of only linear components (in other words, all voltages and currents are related by linear functions), then we can often simplify circuits prior to employing either mesh or nodal analysis. Perhaps the most important result that comes from the knowledge that we are dealing with a completely linear system is that the principle of superposition applies. Given a number of independent sources acting on our circuit, we can add the contribution of each source independently of the other sources. This technique is extremely pervasive throughout the field of engineering, and we will encounter it often. In many real situations, we will find that although several "sources" are acting simultaneously on our "system," typically one of them dominates the system response. Superposition allows us to quickly identify that source, provided that we have a reasonably accurate linear model of the system.

However, from a circuit analysis standpoint, unless we are asked to find which independent source contributes the most to a particular response, we find that rolling up our sleeves and launching straight into either nodal or mesh analysis is often a more straightforward tactic. The reason for this is that applying superposition to a circuit with 12 independent sources will require us to redraw the original circuit 12 times, and often we will have to apply nodal or mesh analysis to each partial circuit, anyway.

The technique of source transformations, however, is often a very useful tool in circuit analysis. Performing source transformations can allow us to consolidate resistors or sources that are not in series or parallel in the original circuit. Source transformations may also allow us to convert all or at least most of the sources in the original circuit to the same type (either all voltage sources or all current sources), so nodal or mesh analysis is more straightforward.

Thévenin's theorem is extremely important for a number of reasons. In working with electronic circuits, we are always aware of the Thévenin equivalent resistance of different parts of our circuit, especially the input and output resistances of amplifier stages. The reason for this is that matching of resistances is frequently the best route to optimizing the performance of a given circuit. We have seen a small preview of this in our discussion of maximum power transfer, where the load resistance should be chosen to match the Thévenin equivalent resistance of the network to which the load is connected. In terms of day-to-day circuit analysis, however, we find that converting part of a circuit to its Thévenin or Norton equivalent is almost as much work as analyzing the complete circuit. Therefore, as in the case of superposition, Thévenin's and Norton's theorems are typically applied only when we require specialized information about part of our circuit.

SUMMARY AND REVIEW

- The principle of superposition states that the *response* in a linear circuit can be obtained by adding the individual responses caused by the separate *independent* sources *acting alone*.
- Superposition is most often used when it is necessary to determine the individual contribution of each source to a particular response.
- A practical model for a real voltage source is a resistor in series with an independent voltage source. A practical model for a real current source is a resistor in parallel with an independent current source.
- Source transformations allow us to convert a practical voltage source into a practical current source, and vice versa.
- Repeated source transformations can greatly simplify analysis of a circuit by providing the means to combine resistors and sources.
- The Thévenin equivalent of a network is a resistor in series with an independent voltage source. The Norton equivalent is the same resistor in parallel with an independent current source.
- There are several ways to obtain the Thévenin equivalent resistance, depending on whether or not dependent sources are present in the network.
- Maximum power transfer occurs when the load resistor matches the Thévenin equivalent resistance of the network to which it is connected.
- When faced with a Δ-connected resistor network, it is straightforward to convert it to a Y-connected network. This can be useful in simplifying the network prior to analysis. Conversely, a Y-connected resistor network can be converted to a Δ-connected network to assist in simplification of the network.

READING FURTHER

A book about battery technology, including characteristics of built-in resistance:

D. Linden, *Handbook of Batteries,* 2nd ed. New York: McGraw-Hill, 1995.

An excellent discussion of pathological cases and various circuit analysis theorems can be found in:

R. A. DeCarlo and P. M. Lin, *Linear Circuit Analysis,* 2nd ed. New York: Oxford University Press, 2001.

EXERCISES

5.1 Linearity and Superposition

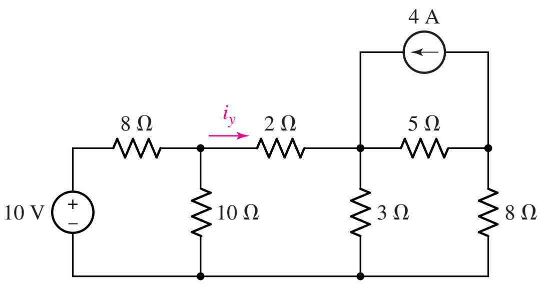
1. The concept of linearity is very important, as linear systems are much more easily analyzed than nonlinear systems. Unfortunately, most practical systems that we encounter are nonlinear in nature. It is possible, however, to create a linear model for a nonlinear system that is valid over a small range of the controlling variable. As an example of this, consider the simple exponential

function e^x. The Taylor series representation of this function is

$$e^x \approx 1 + x + \frac{x^2}{2} + \frac{x^3}{6} + \cdots$$

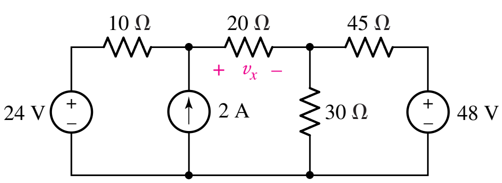
Construct a linear model of this function by truncating it after the linear term (x^1). Evaluate your new function at $x = 0.001$, 0.005, 0.01, 0.05, 0.10, 0.5, 1.0, and 5.0. For which values of x does the linear model give a "reasonable" approximation to e^x?

2. In the circuit of Fig. 5.48, (*a*) determine the contribution of the 4 V source to the current labeled i_1; (*b*) determine the contribution of the 10 V source to i_1; and (*c*) determine i_1.
3. Referring to the two-source circuit depicted in Fig. 5.49, determine the contribution of the 1 A source to v_1, and calculate the total current flowing through the 7 Ω resistor.
4. Employ the principle of superposition to determine the current labeled i_y in the circuit of Fig. 5.50 by considering each source individually.



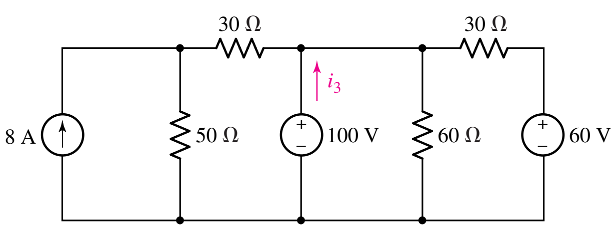
■ **FIGURE 5.50**

5. For the circuit shown in Fig. 5.48, change only the value of the sources to obtain a factor of 10 increase in the current i_1; *both* source values must be changed and *neither* may be set to zero.
6. With sources i_A and v_B on in the circuit of Fig. 5.51 and $v_C = 0$, $i_x = 20$ A; with i_A and v_C on and $v_B = 0$, $i_x = -5$ A; and finally, with all three sources on, $i_x = 12$ A. Find i_x if the only source operating is (*a*) i_A; (*b*) v_B; (*c*) v_C. (*d*) Find i_x if i_A and v_C are doubled in amplitude and v_B is reversed.
7. Use superposition to find the value of v_x in the circuit of Fig. 5.52.

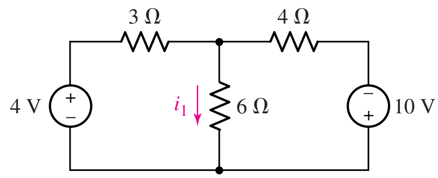


■ **FIGURE 5.52**

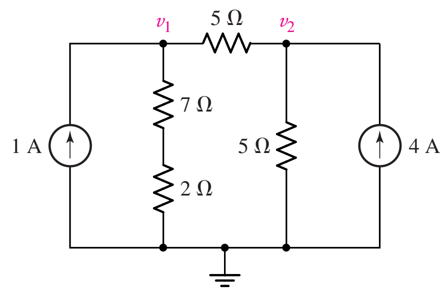
8. Apply superposition to the circuit of Fig. 5.53 to find i_3.



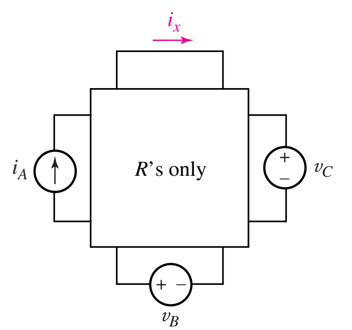
■ **FIGURE 5.53**



■ **FIGURE 5.48**



■ **FIGURE 5.49**



■ **FIGURE 5.51**

9. (*a*) Use the superposition theorem to find i_2 in the circuit shown in Fig. 5.54. (*b*) Calculate the power absorbed by each of the five circuit elements.

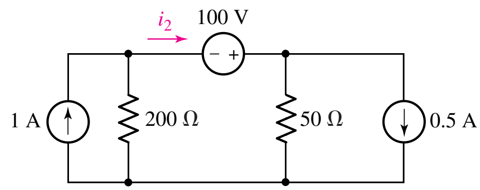


FIGURE 5.54

10. Use superposition on the circuit shown in Fig. 5.55 to find the voltage v. Note that there is a dependent source present.

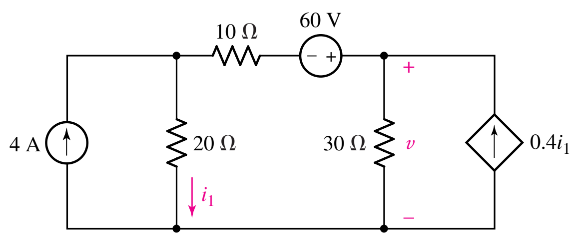


FIGURE 5.55

11. In the circuit shown in Fig. 5.56: (*a*) if $i_A = 10$ A and $i_B = 0$, then $v_3 = 80$ V; find v_3 if $i_A = 25$ A and $i_B = 0$. (*b*) If $i_A = 10$ A and $i_B = 25$ A, then $v_4 = 100$ V, while $v_4 = -50$ V if $i_A = 25$ A and $i_B = 10$ A; find v_4 if $i_A = 20$ A and $i_B = -10$ A.

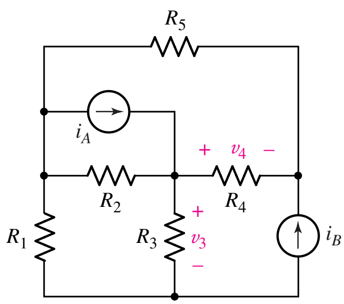


FIGURE 5.56

12. Use superposition to determine the voltage across the current source in Fig. 5.57.

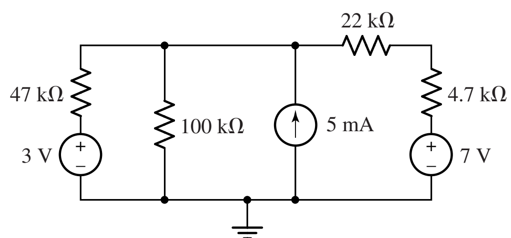


FIGURE 5.57

13. Use superposition to find the power dissipated by the 500 kΩ resistor in Fig. 5.58.

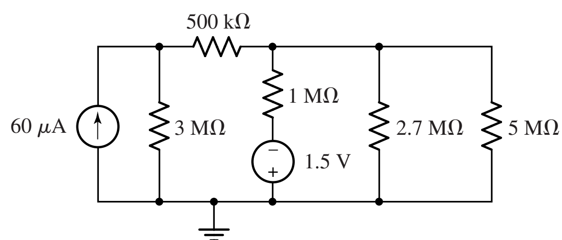
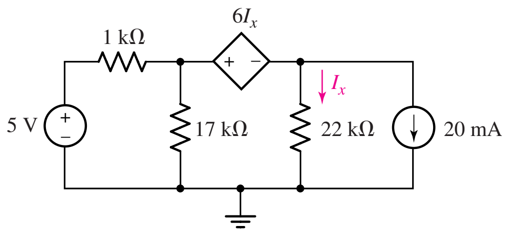


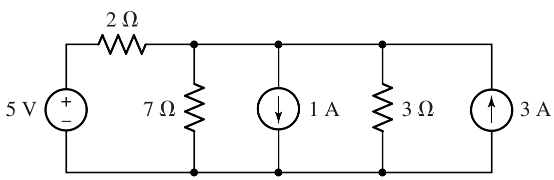
FIGURE 5.58

14. Employ superposition to determine the voltage across the 17 kΩ resistor in Fig. 5.59. If the maximum power rating of the resistor is 250 mW, what is the maximum positive voltage to which the 5 V source can be increased before the resistor overheats?



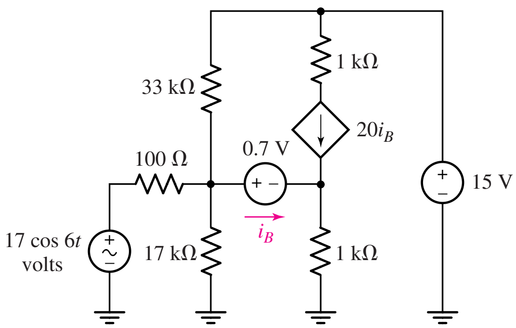
■ FIGURE 5.59

15. Which source in Fig. 5.60 contributes the most to the power dissipated in the 2 Ω resistor? The least? What *is* the power dissipated in the 2 Ω resistor?



■ FIGURE 5.60

16. Use superposition to find i_B in the circuit of Fig. 5.61, which is a commonly used model circuit for a bipolar junction transistor amplifier.

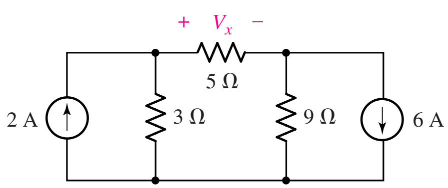


■ FIGURE 5.61

17. For the circuit shown in Fig. 5.62,

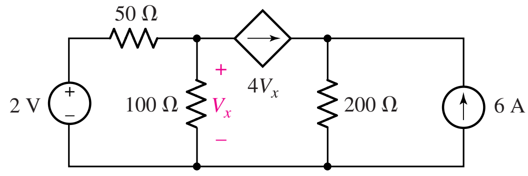
(*a*) Use superposition to compute V_x.

(*b*) Verify the contribution of each source to V_x using a dc sweep PSpice analysis. Submit a labeled schematic, relevant probe output, and a brief summary of the results.



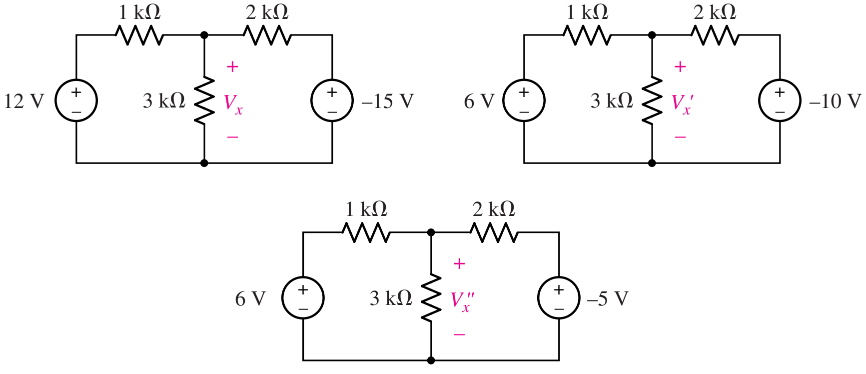
■ FIGURE 5.62

18. For the circuit shown in Fig. 5.63,
    (*a*) Use superposition to compute V_x.
    (*b*) Verify the contribution of each source to V_x using a dc sweep PSpice analysis. Submit a labeled schematic, relevant probe output, and a brief summary of the results.



■ **FIGURE 5.63**

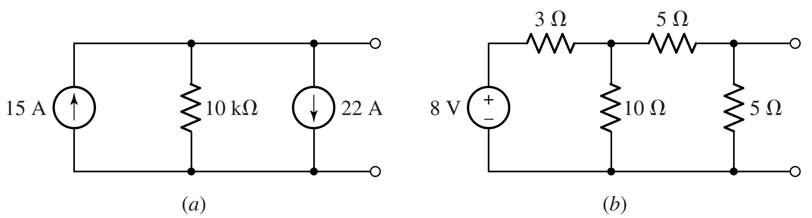
19. Consider the three circuits shown in Fig. 5.64. Analyze each circuit, and demonstrate that $V_x = V_x' + V_x''$ (i.e., *superposition is most useful when sources are set to zero, but the principle is in fact much more general than that*).



■ **FIGURE 5.64**

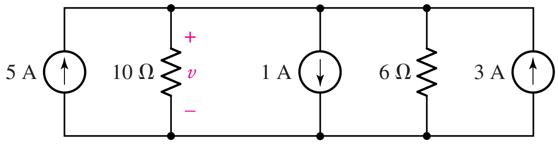
5.2 Source Transformations

20. With the assistance of the method of source transformations, (*a*) convert the circuit of Fig. 5.65*a* to a single independent voltage source in series with an appropriately sized resistor; and (*b*) convert the circuit of Fig. 5.65*b* to a single independent current source in parallel with an appropriately sized resistor. For both (*a*) and (*b*), leave the right-hand terminals in your final circuit.



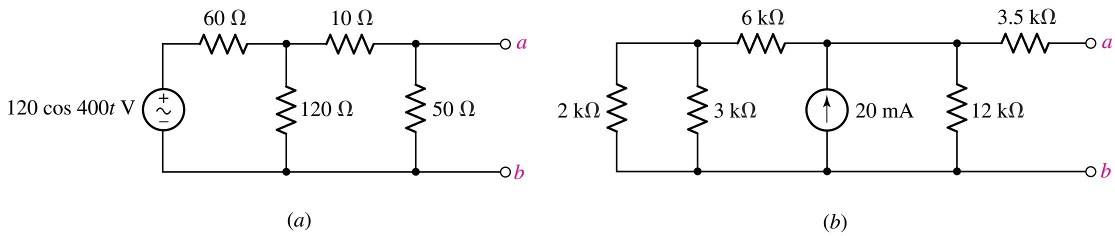
■ **FIGURE 5.65**

21. (*a*) Use the method of source transformations to reduce the circuit of Fig. 5.66 to a practical voltage source in series with the 10 Ω resistor. (*b*) Calculate v. (*c*) Explain why the 10 Ω resistor should not be included in a source transformation.



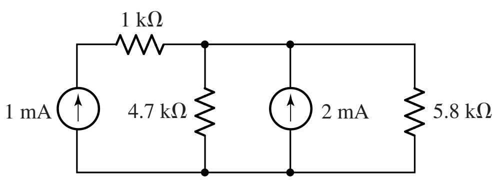
■ **FIGURE 5.66**

22. Use source transformations and resistance combinations to simplify both of the networks of Fig. 5.67 until only two elements remain to the left of terminals *a* and *b*.



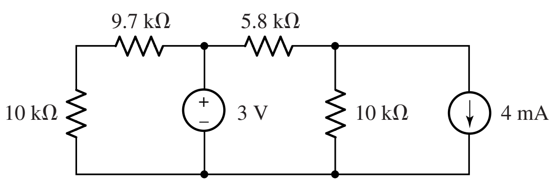
■ **FIGURE 5.67**

23. Using source transformation to first simplify the circuit, determine the power dissipated by the 5.8 kΩ resistor in Fig. 5.68.



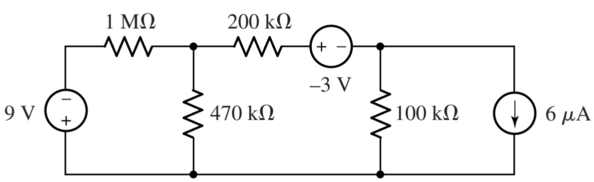
■ **FIGURE 5.68**

24. Using source transformation, determine the power dissipated by the 5.8 kΩ resistor in Fig. 5.69.



■ **FIGURE 5.69**

25. Determine the power dissipated by the 1 MΩ resistor using source transformation to first simplify the circuit shown in Fig. 5.70.



■ **FIGURE 5.70**

26. Determine I_1 using source transformation to first simplify the circuit of Fig. 5.71.

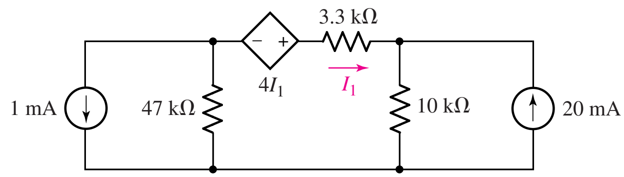


FIGURE 5.71

27. (*a*) Find V_1 in the circuit of Fig. 5.72 using source transformation to obtain a simplified equivalent circuit first.

(*b*) Verify your analysis by performing a PSpice analysis of the circuit in Fig. 5.72. Submit a schematic with V_1 clearly labeled.

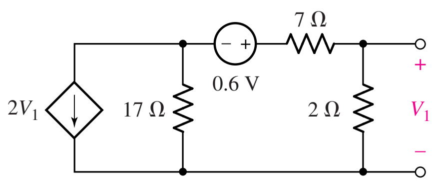


FIGURE 5.72

28. (*a*) Use repeated source transformation to determine the current I_x as indicated in Fig. 5.73.

(*b*) Verify your analysis by performing a PSpice analysis of the circuit in Fig. 5.73. Submit a schematic with I_X clearly labeled.

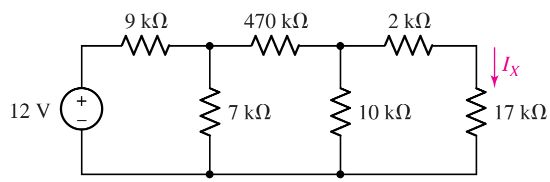


FIGURE 5.73

29. Use repeated source transformation to determine the current I_X in the circuit of Fig. 5.74.

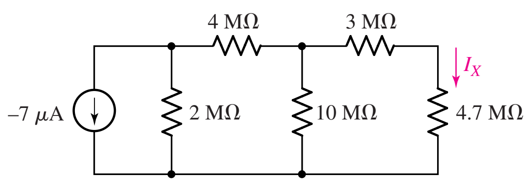


FIGURE 5.74

30. Convert the circuit in Fig. 5.75 to a single current source in parallel with a single resistor.

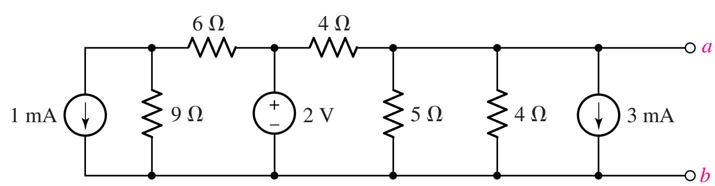
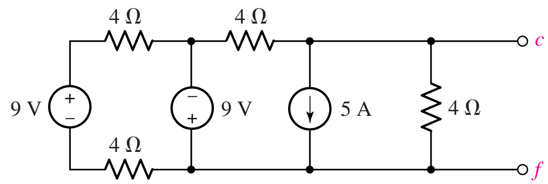
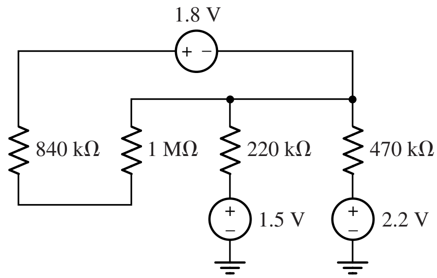


FIGURE 5.75

31. Use source transformation to convert the circuit in Fig. 5.76 to a single current source in parallel with a single resistor.



■ **FIGURE 5.76**



■ **FIGURE 5.77**

32. Determine the power dissipated by the 1 MΩ resistor in the circuit of Fig. 5.77.

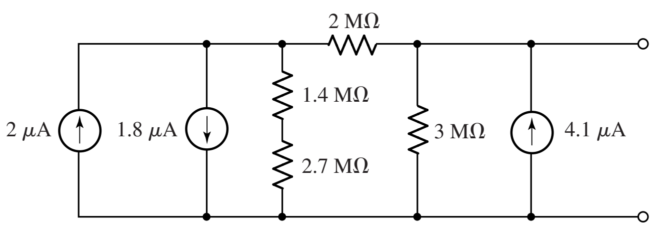
33. The measurements in Table 5.1 are made on a 1.5 V alkaline battery. Use the information to construct a simple two-component practical voltage source model for the battery that is relatively accurate for currents in the range of 1 to 20 mA. Note that besides the obvious experimental error, the "internal resistance" of the battery is significantly different over the current range measured in the experiment.

TABLE **5.1** Measured current-voltage characteristics of a 1.5 V alkaline battery connected to a variable load resistance

Current Output (mA)	Terminal Voltage (V)
0.0000589	1.584
0.3183	1.582
1.4398	1.567
7.010	1.563
12.58	1.558

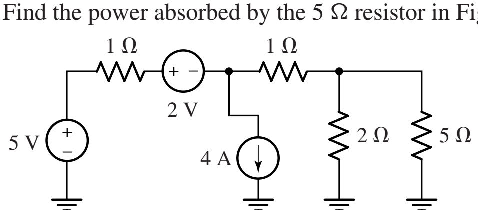
34. Use the data in Table 5.1 to construct a simple two-component practical current source model for the battery that is relatively accurate for currents in the range of 1 to 20 mA. Note that besides the obvious experimental error, the "internal resistance" of the battery is significantly different over the current range measured in the experiment.

35. Reduce the circuit in Fig. 5.78 to a single voltage source in series with a single resistor. Leave the right-hand terminals in your final circuit.



■ **FIGURE 5.78**

36. Find the power absorbed by the 5 Ω resistor in Fig. 5.79.



■ **FIGURE 5.79**

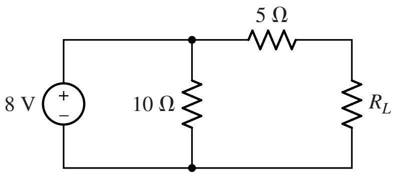


FIGURE 5.80

37. (*a*) Convert the circuit of Fig. 5.80 to a practical current source in parallel with R_L. (*b*) Verify your answer using PSpice and a value of 5 Ω for R_L. Submit properly labeled schematics for each circuit, with the voltage across R_L clearly identified.

38. (*a*) Reduce the circuit of Fig. 5.81 as much as possible, transform the two voltage sources into current sources, then compute the power dissipated in the top 5 Ω resistor. (*b*) Verify your answer by simulating both circuits with PSpice. Submit a properly labeled schematic for each circuit, with the power dissipated in the resistor of interest clearly identified. (*c*) Does the value of the 1 Ω or the 7 Ω resistor affect your answer in any way? Explain why or why not.

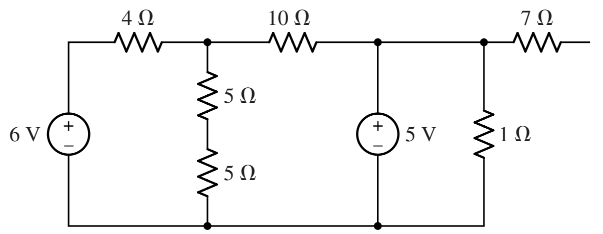


FIGURE 5.81

39. For the circuit of Fig. 5.82, convert all sources (both dependent and independent) to current sources, combine the dependent sources, and calculate the voltage v_3.

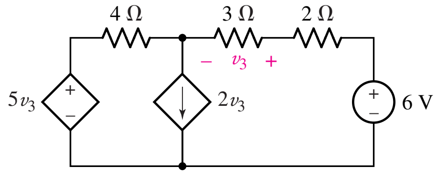


FIGURE 5.82

5.3 Thévenin and Norton Equivalent Circuits

40. (*a*) Find the Thévenin equivalent at terminals *a* and *b* for the network shown in Fig. 5.83. How much power would be delivered to a resistor connected to *a* and *b* if R_{ab} equals (*b*) 50 Ω; (*c*) 12.5 Ω?

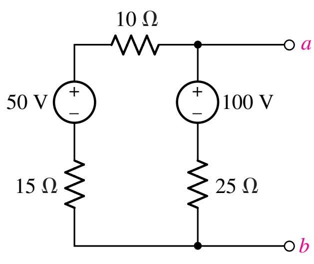


FIGURE 5.83

41. (*a*) Employ Thévenin's theorem to simplify the network connected to the 5 Ω resistor of Fig. 5.84. (*b*) Compute the power absorbed by the 5 Ω resistor using your simplified circuit. (*c*) Verify your solution with PSpice. Submit a properly labeled schematic for each circuit with the requested power quantity clearly identified.

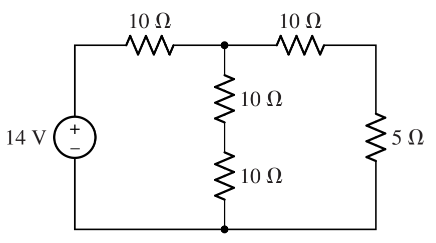


FIGURE 5.84

42. (*a*) Find the Thévenin equivalent of the network connected to the 7 Ω resistor of Fig. 5.85. (*b*) Find the Norton equivalent of the network connected to the 7 Ω resistor of Fig. 5.85. (*c*) Compute the voltage v_1 using both of your equivalent circuits. (*d*) Replace the 7 Ω resistor with a 1 Ω resistor, and recompute v_1 using either circuit.

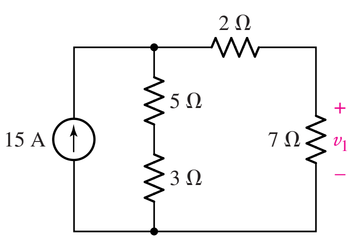
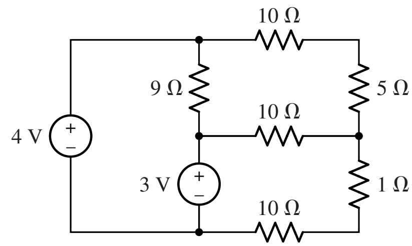


FIGURE 5.85

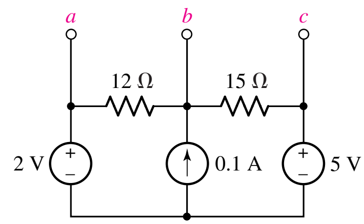
43. (*a*) A tungsten-filament light bulb is connected to a 10 mV test voltage, and a current of 400 μA is measured. What is the Thévenin equivalent of the light bulb? (*b*) The bulb is now connected to a 110 V supply, and a current of 363.6 mA is measured. Determine the Thévenin equivalent based on this measurement. (*c*) Why is the Thévenin equivalent of the light bulb apparently dependent on the test conditions, and what implications does this have if we need to analyze a circuit containing the bulb?

44. (*a*) Find both the Thévenin and Norton equivalents for the network connected to the 1 Ω resistor of Fig. 5.86. (*b*) Compute the power absorbed by the 1 Ω resistor using both equivalent circuits. (*c*) Verify using PSpice. Submit a properly labeled schematic for each of the three circuits, with the requested power quantity clearly identified.



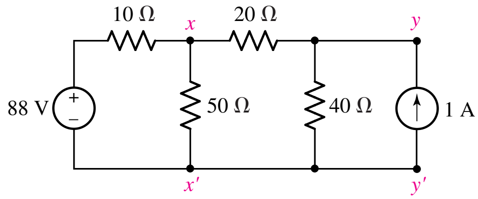
■ **FIGURE 5.86**

45. For the network of Fig. 5.87: (*a*) remove terminal *c* and find the Norton equivalent seen at terminals *a* and *b*; (*b*) repeat for terminals *b* and *c* with *a* removed.



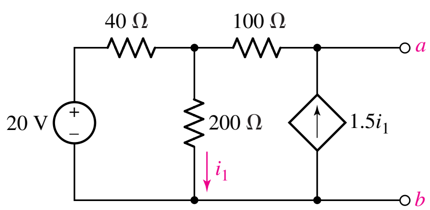
■ **FIGURE 5.87**

46. Find the Thévenin equivalent of the network in Fig. 5.88 as viewed from terminals: (*a*) x and x'; (*b*) y and y'.



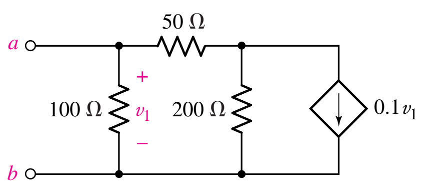
■ **FIGURE 5.88**

47. (*a*) Find the Thévenin equivalent of the network shown in Fig. 5.89. (*b*) What power would be delivered to a load of 100 Ω at *a* and *b*?



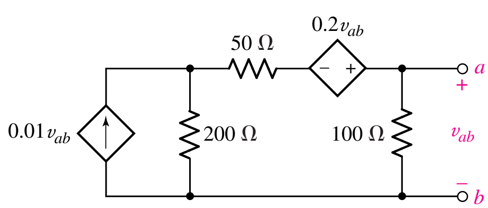
■ **FIGURE 5.89**

48. Find the Norton equivalent of the network shown in Fig. 5.90.



■ **FIGURE 5.90**

49. Find the Thévenin equivalent of the two-terminal network shown in Fig. 5.91.



■ **FIGURE 5.91**

50. Find the Thévenin equivalent of the circuit in Fig. 5.92.

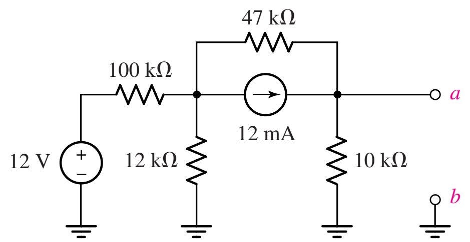


FIGURE 5.92

51. For the network in Fig. 5.93, determine: (*a*) the Thévenin equivalent; (*b*) the Norton equivalent.

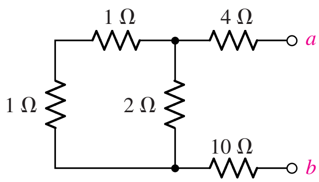


FIGURE 5.93

52. For the circuit in Fig. 5.94*a*, determine the Norton equivalent of the network connected to R_L. For the circuit in Fig. 5.94*b*, determine the Thévenin equivalent of the network connected to R_L.

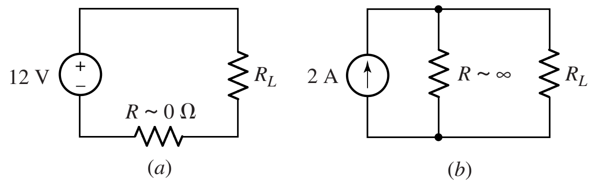


FIGURE 5.94

53. Determine the Thévenin and Norton equivalents of the network shown in Fig. 5.95.

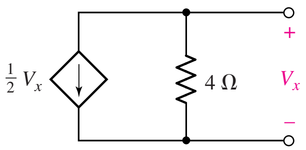


FIGURE 5.95

54. Determine the Thévenin and Norton equivalents of the network shown in Fig. 5.96.

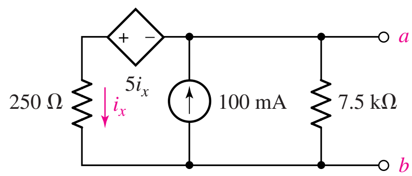


FIGURE 5.96

55. Determine the Thévenin and Norton equivalents of the network shown in Fig. 5.97.

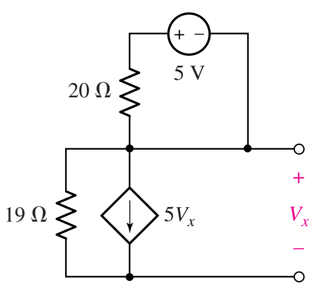


FIGURE 5.97

56. Find the Thévenin equivalent resistance seen by the 2 kΩ resistor in the circuit of Fig. 5.98. Ignore the dashed line in the figure.

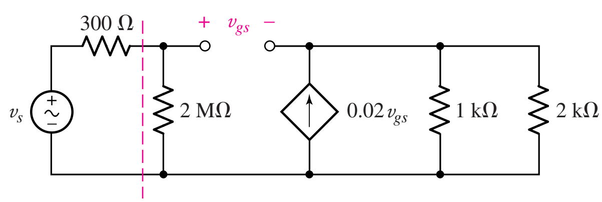
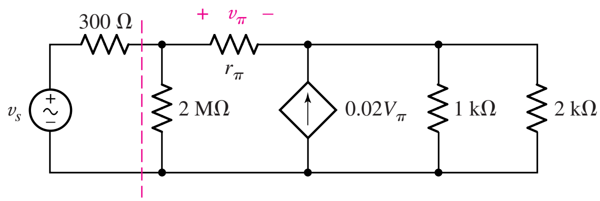


FIGURE 5.98

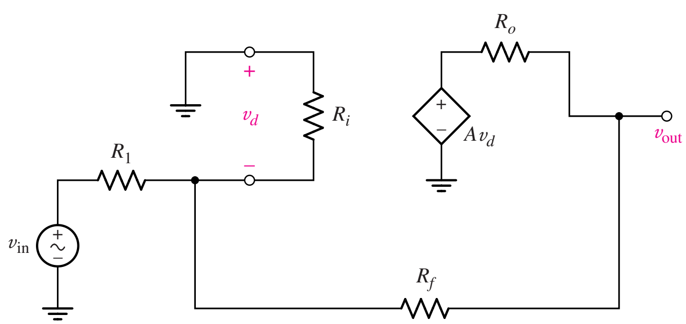
57. Referring to the circuit of Fig. 5.98, determine the Thévenin equivalent resistance of the circuit to the right of the dashed line. This circuit is a common-source transistor amplifier, and you are calculating its input resistance.

58. Referring to the circuit of Fig. 5.99, determine the Thévenin equivalent resistance of the circuit to the right of the dashed line. This circuit is a common-collector transistor amplifier, and you are calculating its input resistance.



■ **FIGURE 5.99**

59. The circuit shown in Fig. 5.100 is a reasonably accurate model of an operational amplifier. In cases where R_i and A are very large and $R_o \sim 0$, a resistive load (such as a speaker) connected between ground and the terminal labeled v_{out} will see a voltage $-R_f/R_1$ times larger than the input signal v_{in}. Find the Thévenin equivalent of the circuit, taking care to label v_{out}.

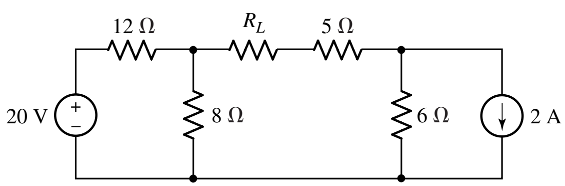


■ **FIGURE 5.100**

5.4 Maximum Power Transfer

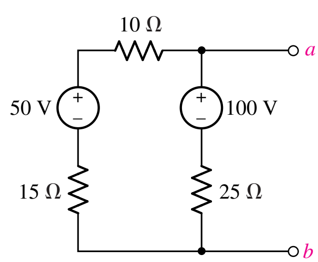
60. Assuming that we can determine the Thévenin equivalent resistance of our wall socket, why don't toaster, microwave oven, and TV manufacturers match each appliance's Thévenin equivalent resistance to this value? Wouldn't it permit maximum power transfer from the utility company to our household appliances?

61. If any value whatsoever may be selected for R_L in the circuit of Fig. 5.101, what is the maximum power that could be dissipated in R_L?



■ **FIGURE 5.101**

62. (*a*) Find the Thévenin equivalent at terminals *a* and *b* for the network shown in Fig. 5.102. How much power would be delivered to a resistor connected to *a* and *b* if R_{ab} equals (*b*) 10 Ω; (*c*) 75 Ω?



■ **FIGURE 5.102**

63. (*a*) Determine the Thévenin equivalent of the network shown in Fig. 5.103, and (*b*) find the maximum power that can be drawn from it.

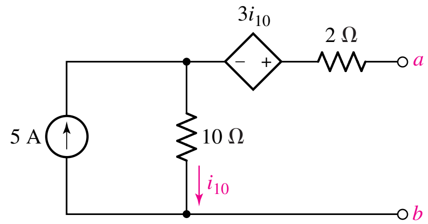


FIGURE 5.103

64. With reference to the circuit of Fig. 5.104: (*a*) determine that value of R_L to which a maximum power can be delivered, and (*b*) calculate the voltage across R_L then (+ reference at top).

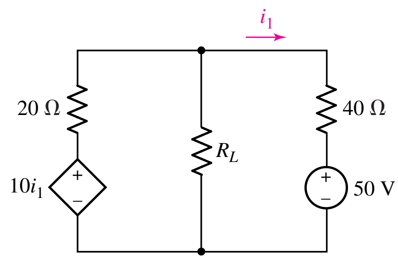


FIGURE 5.104

65. A certain practical dc voltage source can provide a current of 2.5 A when it is (momentarily) short-circuited, and can provide a power of 80 W to a 20 Ω load. Find (*a*) the open-circuit voltage and (*b*) the maximum power it could deliver to a well-chosen R_L. (*c*) What is the value of that R_L?

66. A practical current source provides 10 W to a 250 Ω load and 20 W to an 80 Ω load. A resistance R_L, with voltage v_L and current i_L, is connected to it. Find the values of R_L, v_L, and i_L if (*a*) $v_L i_L$ is a maximum; (*b*) v_L is a maximum; (*c*) i_L is a maximum.

67. A certain battery can accurately be modeled as a 9 V independent source in series with a 1.2 Ω resistor over the current range of interest. No current flows if an infinite resistance load is connected to the battery. We also know that maximum power will be transferred to a resistor of 1.2 Ω, and less power transferred to either a 1.1 Ω or 1.3 Ω resistor. However, if we simply short the terminals of the battery together (not recommended!), we will obtain *much more* current than for a 1.2 Ω resistive load. Doesn't this conflict with what we derived previously for maximum power transfer (*after all, isn't power proportional to* i^2)? Explain.

68. The circuit in Fig. 5.105 is part of an audio amplifier. If we want to transfer maximum power to the 8 Ω speaker, what value of R_E is needed? Verify your solution with PSpice.

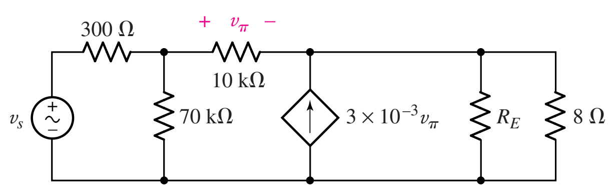


FIGURE 5.105

69. The circuit shown in Fig. 5.106 depicts a circuit separated into two stages. Select R_1 so that maximum power is transferred from stage 1 to stage 2.

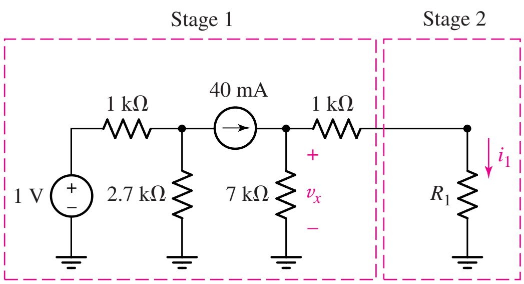


FIGURE 5.106

5.5 Delta-Wye Conversion

70. Convert the network in Fig. 5.107 to a Y-connected network.

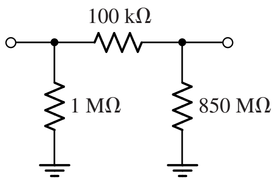


FIGURE 5.107

71. Convert the network in Fig. 5.108 to a Δ-connected network.

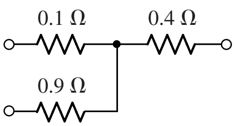


FIGURE 5.108

72. Find R_{in} for the network shown in Fig. 5.109.

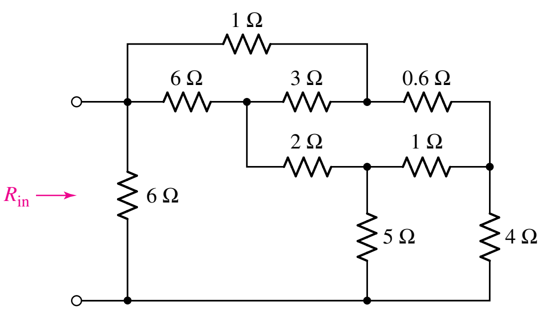


FIGURE 5.109

73. Use Y-Δ and Δ-Y transformations to find the input resistance of the network shown in Fig. 5.110.

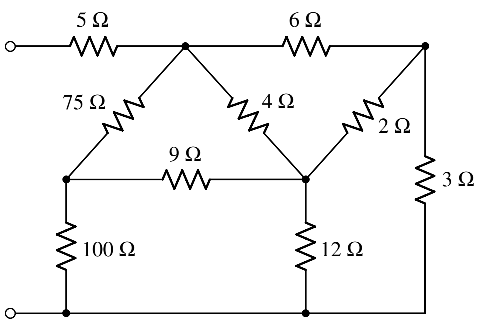
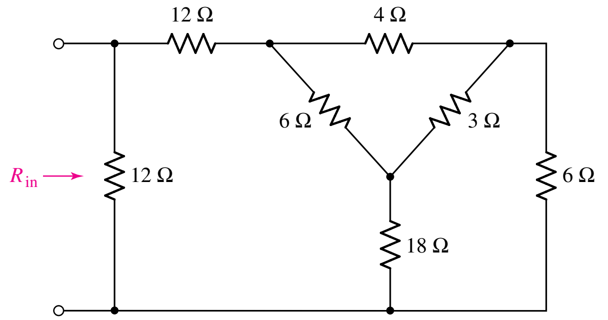


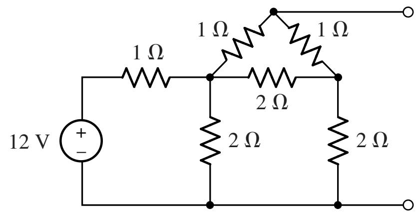
FIGURE 5.110

74. Find R_{in} for the circuit of Fig. 5.111.



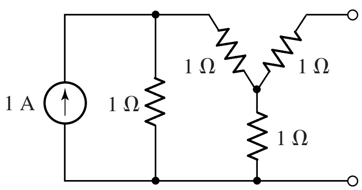
■ **FIGURE 5.111**

75. Find the Thévenin equivalent of the circuit in Fig. 5.112.



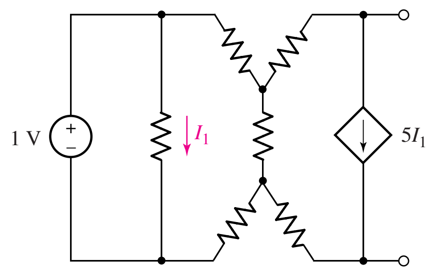
■ **FIGURE 5.112**

76. Find the Norton equivalent of the circuit in Fig. 5.113.



■ **FIGURE 5.113**

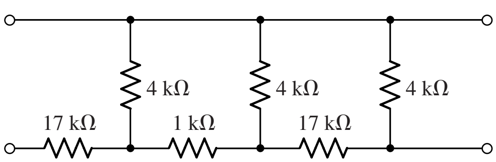
77. If all resistors in Fig. 5.114 are 10 Ω, determine the Thévenin equivalent for the circuit.



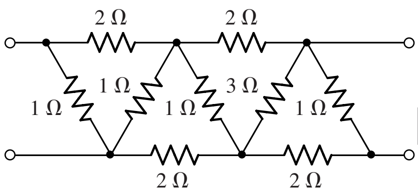
■ **FIGURE 5.114**

78. (*a*) Replace the network in Fig. 5.115 with an equivalent three-resistor Y network.

(*b*) Perform a PSpice analysis to verify that your answer is in fact equivalent. (Hint: Try adding a load resistor.)



■ **FIGURE 5.115**



■ **FIGURE 5.116**

79. (*a*) Replace the network in Fig. 5.116 with an equivalent three-resistor Δ network.

(*b*) Perform a PSpice analysis to verify that your answer is in fact equivalent. (Hint: Try adding a load resistor.)

5.6 Selecting an Approach: A Summary of Various Techniques

80. The circuit shown in Fig. 5.117 is a reasonably accurate model for a bipolar junction transistor operating in what is known as the *forward active region*. Determine the collector current I_C. Verify your answer with PSpice.

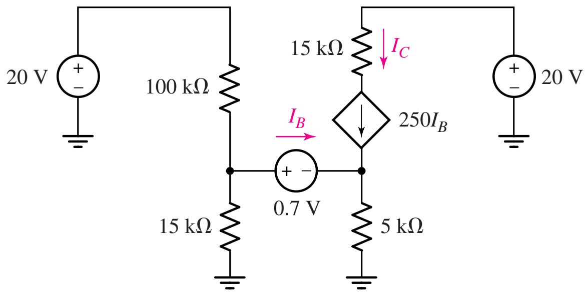


FIGURE 5.117

81. The load resistor in Fig. 5.118 can safely dissipate up to 1 W before overheating and bursting into flame. The lamp can be treated as a 10.6 Ω resistor if less than 1 A flows through it and a 15 Ω resistor if more than 1 A flows through it. What is the maximum permissible value of I_S? Verify your answer with PSpice.

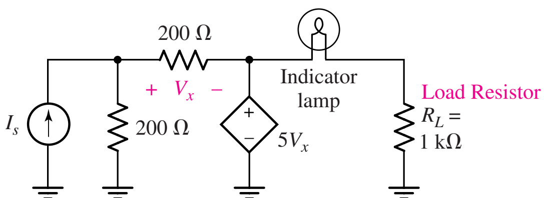


FIGURE 5.118

82. The human ear can detect sound waves in the frequency range of about 20 Hz to 20 kHz. If each 8 Ω resistor in Fig. 5.119 is a loudspeaker, which of the signal generators (modeled as practical voltage sources) produces the most sound? (Take "loudness" as proportional to power delivered to a speaker.)

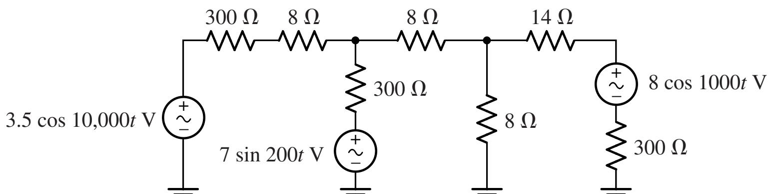


FIGURE 5.119

83. A DMM is connected to a resistor circuit as shown in Fig. 5.120. If the input resistance of the DMM is 1 MΩ, what value will be displayed if the DMM is measuring resistance?

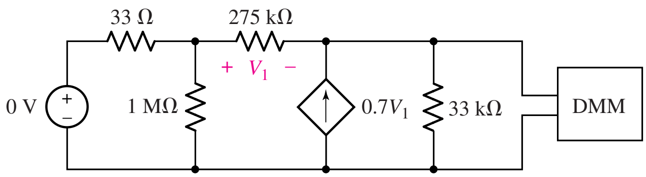


FIGURE 5.120

84. A metallic substance is extracted from a meteorite found in rural Indiana. The substance is found to have a resistivity of 50 Ω · cm and is fabricated into a simple cylinder. The cylinder is connected into the circuit of Fig. 5.121 and is found to have a temperature dependence of $T = 200P^{0.25}$ °C, where P is the power delivered to the cylinder in watts. Interestingly enough, the resistivity of the substance does not appear to depend on temperature. If $R = 10\ \Omega$ and is absorbing maximum power in the circuit shown, what is the temperature of the cylinder?

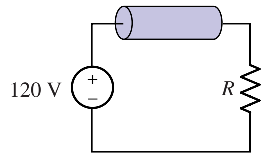


FIGURE 5.121

D 85. As part of a security system, a very thin 100 Ω wire is attached to a window using nonconducting epoxy. Given only a box of twelve rechargeable 1.5 V AAA batteries, one thousand 1 Ω resistors, and a 2900 Hz piezo buzzer that draws 15 mA at 6 V, design a circuit with no moving parts that will set off the buzzer if the window is broken (and hence the thin wire as well). Note that the buzzer requires a dc voltage of at least 6 V (maximum 28 V) to operate.

D 86. Three 45 W light bulbs originally wired in a Y network configuration with a 120 V ac source connected across each port are rewired as a Δ network. The neutral, or center, connection is not used. If the intensity of each light is proportional to the power it draws, design a new 120 V ac power circuit so that the three lights have the same intensity in the Δ configuration as they did when connected in a Y configuration. Verify your design using PSpice by comparing the power drawn by each light in your circuit (modeled as an appropriately chosen resistor value) with the power each would draw in the original Y-connected circuit.

D 87. A certain red LED has a maximum current rating of 35 mA, and if this value is exceeded, overheating and catastrophic failure will result. The resistance of the LED is a nonlinear function of its current, but the manufacturer warrants a minimum resistance of 47 Ω and a maximum resistance of 117 Ω. Only 9 V batteries are available to power the LED. Design a suitable circuit to deliver the maximum power possible to the LED without damaging it. Use only combinations of the standard resistor values given in the inside front cover.