RESISTOR NETWORKS

MS424 Lecture Note

In the previous lectures

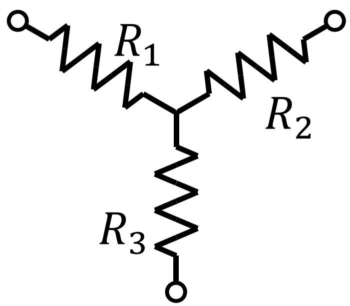
- Charge, current, voltage, energy, power
- Sources
- Simplifying source networks
- Ohm’s law, KCL, KVL
- Series and parallel resistors

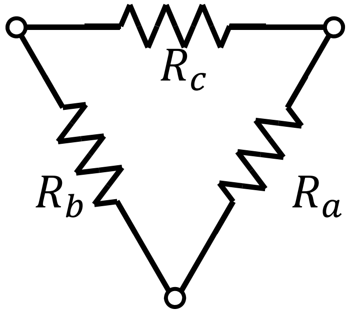
Today

- Resistor networks
  - Series resistors
  - Parallel resistors
  - Wye-Delta transformation

1 + 1 = 2

1 + 1 = 0.5





- Methods of analysis
  - Nodal analysis

Find v_n using KCL

Resistor Networks

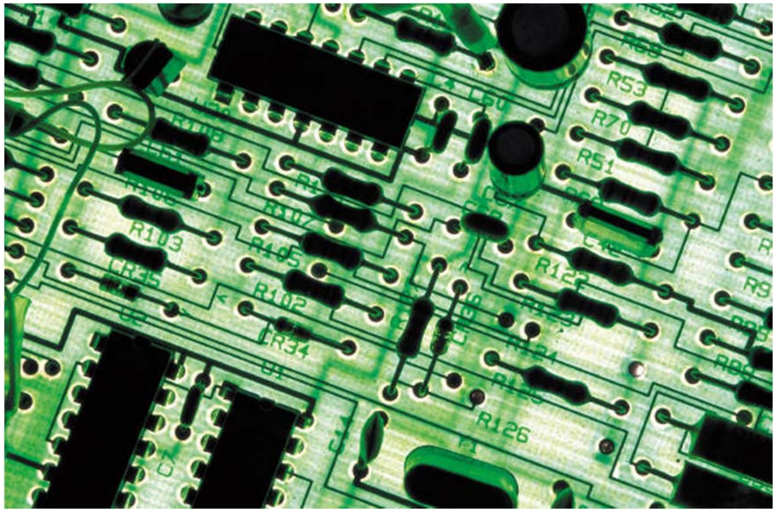
Series Resistors and Voltage Division

Parallel Resistors and Current Division

Wye-Delta Transformation

Role of resistors

- Regulate currents (c.f. hole and pipes in water clocks)
- Control temporal response
  (in combination with capacitors or inductors)
- Convert electric energy to another form of energy
  (heat, light, sound, etc.)

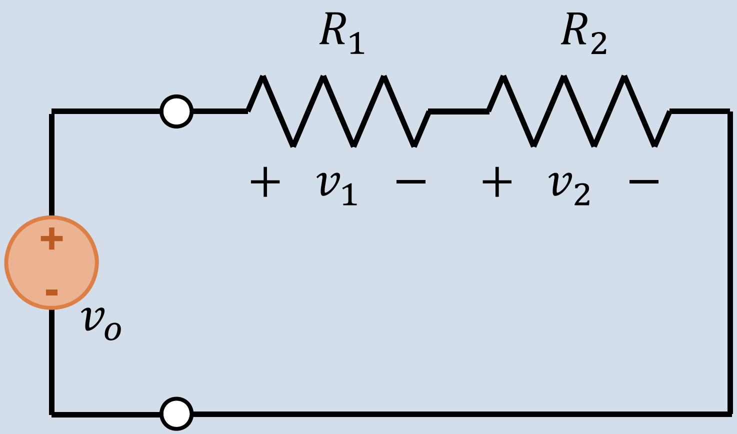


Resistor Network

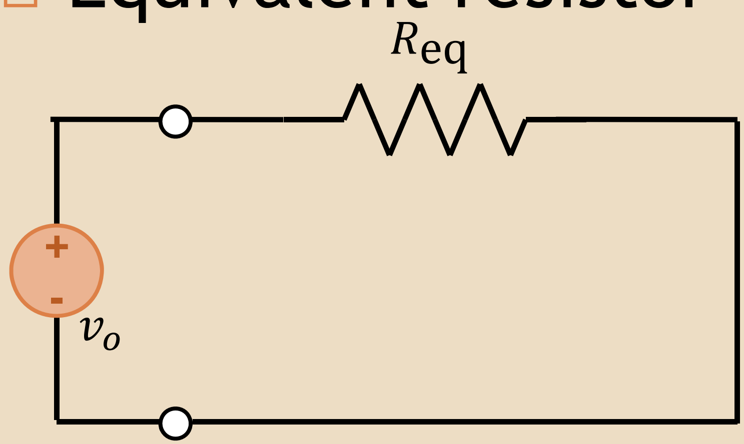
- In actual circuits, many resistors are placed in various topologies
  - To regulate currents in different branches, or
  - To form a new "effective" resistor from a limited set of available resistors, or
  - Due to other requirements
- Complicated resistor network can be simplified using a few rules

Series resistors and voltage division

- Series resistors



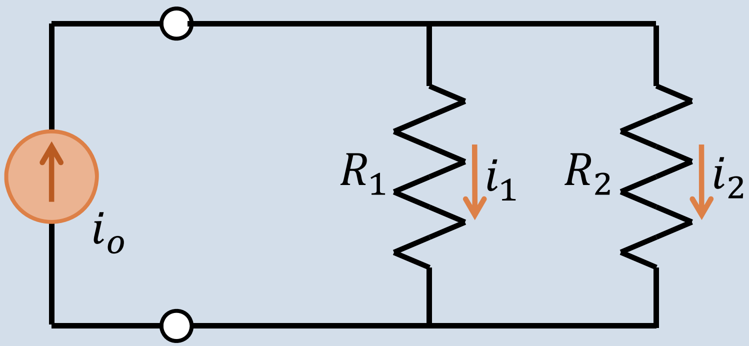
- Equivalent resistor



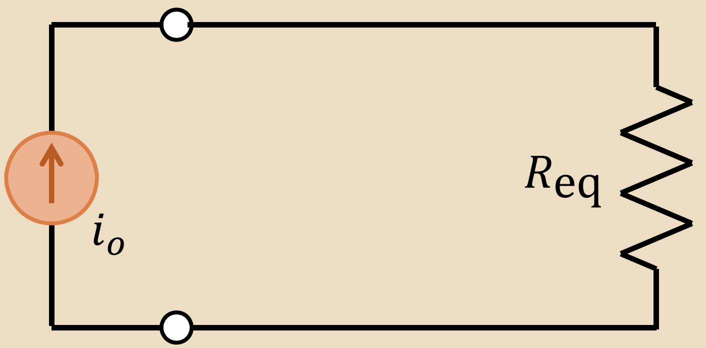
- $$v_1 = \frac{R_1}{R_1+R_2} v_o,$$
  $$v_2 = \frac{R_2}{R_1+R_2} v_o,$$
  $$R_{\mathrm{eq}} = R_1 + R_2.$$
- In general,
  $$v_m = \frac{R_m}{R_{\mathrm{eq}}} v_o,$$
  $$R_{\mathrm{eq}} = \sum R_m.$$

Parallel resistors and current division

- Parallel resistors



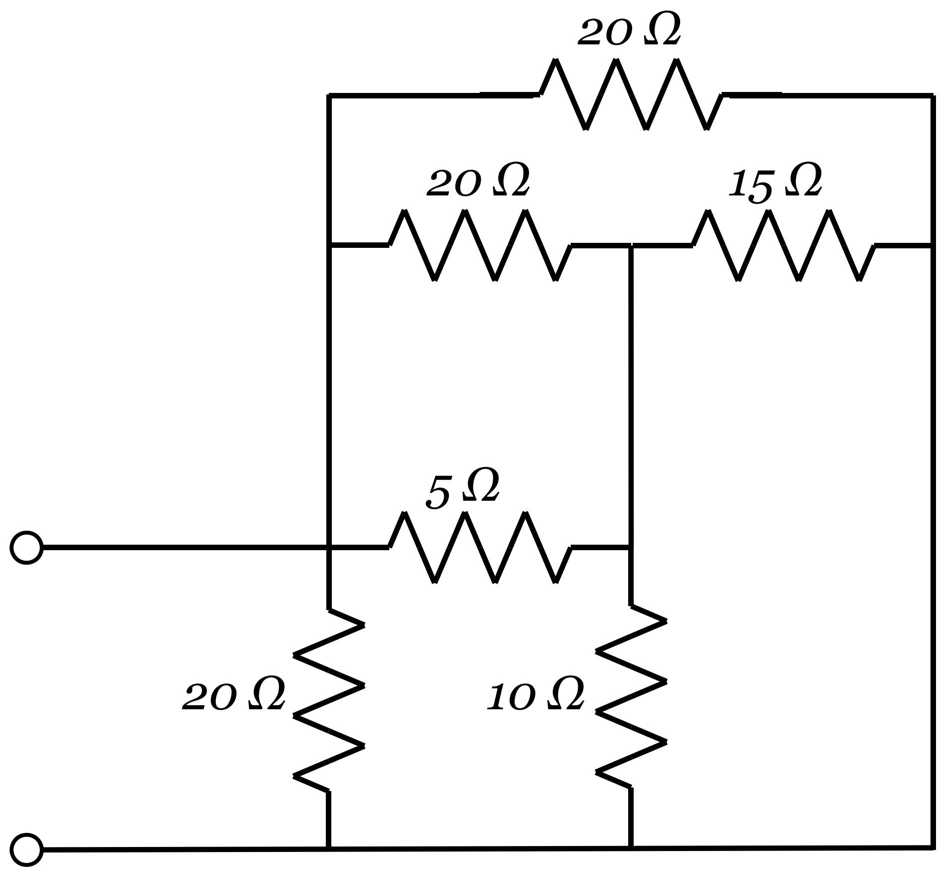
- Equivalent resistor



- $i_1 = \frac{G_1}{G_1+G_2} i_o = \frac{R_2}{R_1+R_2} i_o,$
  $i_2 = \frac{G_2}{G_1+G_2} i_o = \frac{R_1}{R_1+R_2} i_o,$
  $G_{eq} = G_1 + G_2,$
  $R_{eq} = R_1 R_2/(R_1 + R_2).$
- In general,
  $i_n = \frac{G_n}{G_{eq}} i_o,$
  $G_{eq} = \sum G_n,$
  $\frac{1}{R_{eq}} = \sum \frac{1}{R_n}.$

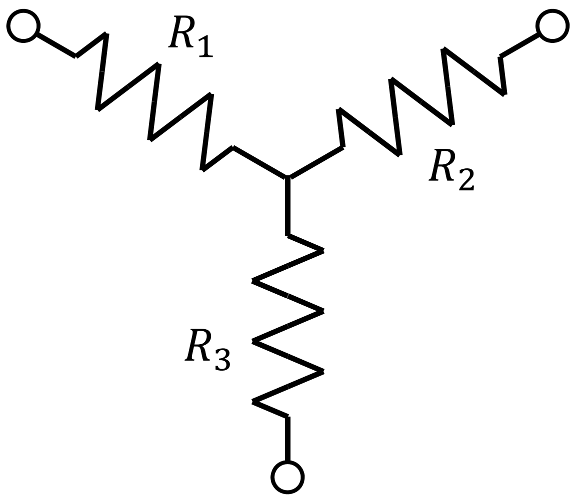
Quiz

Simplify the above resistor network (What is the equivalent resistance?)

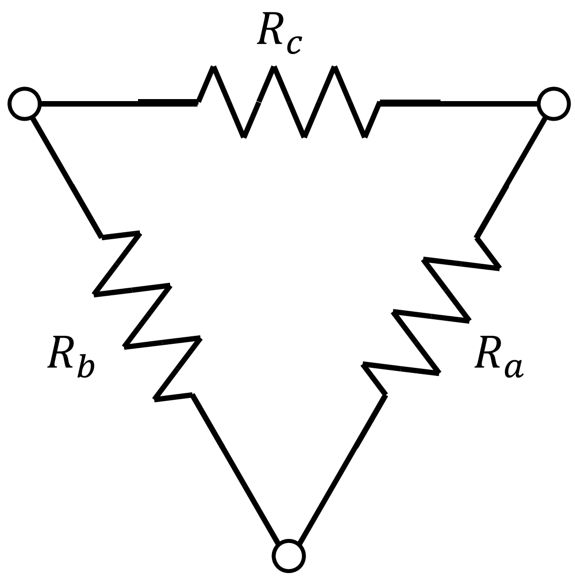


Wye and Delta networks

Wye network

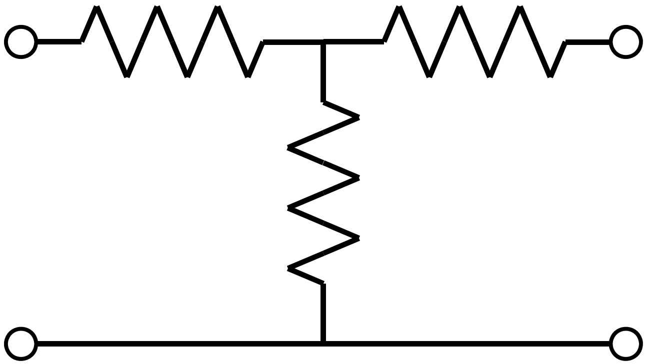


Delta network



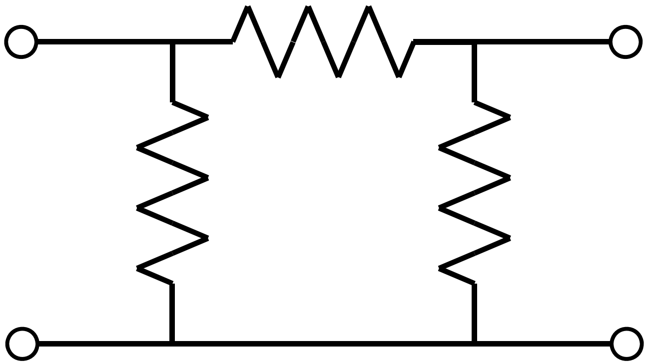
Tee and Pi networks

Tee network



Same as Wye

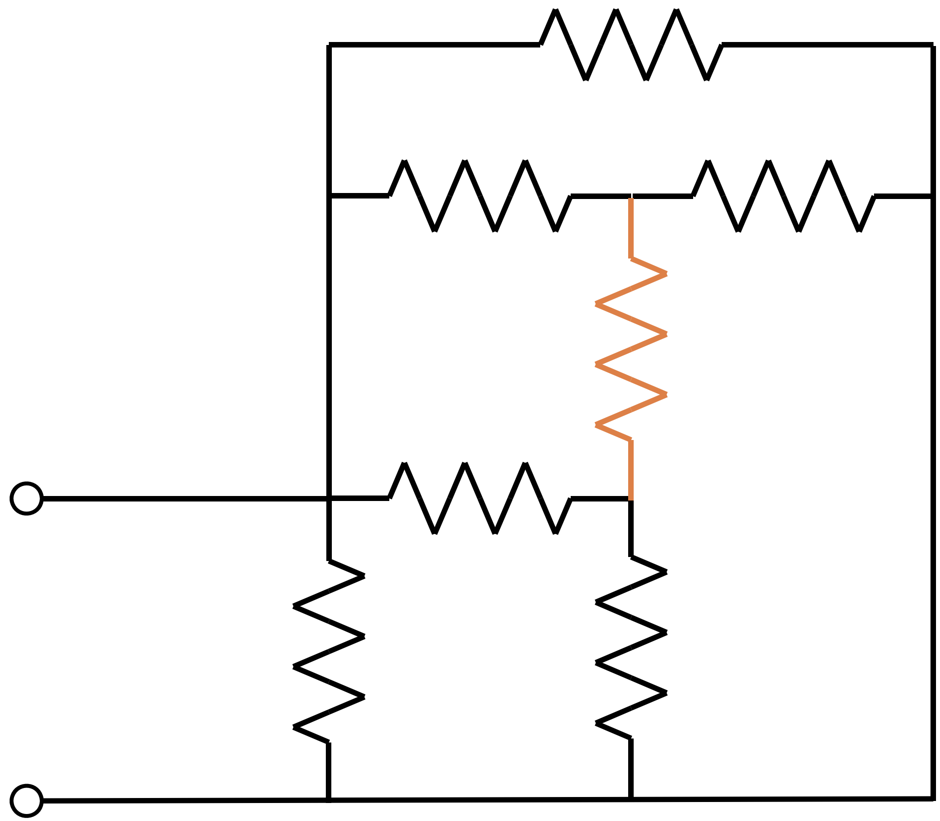
Pi network



Same as Delta

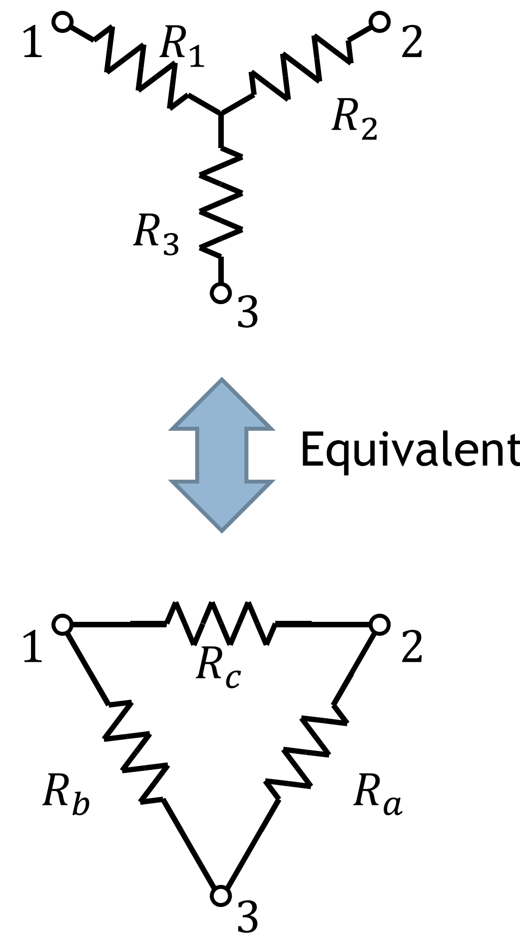
12 Quiz

Can you simplify the above network
based on the rules for series and parallel resistors only?

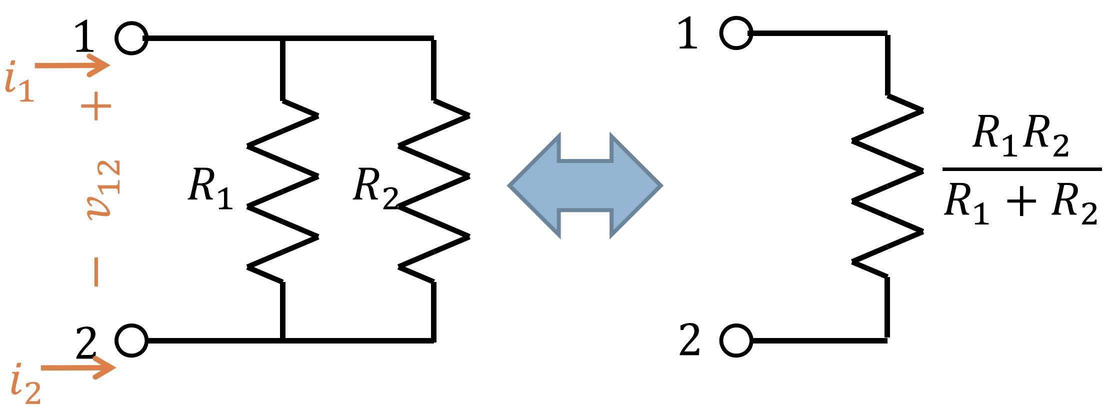


Wye-Delta transformation

- There exists an equivalent Delta network for every Wye network and vice versa.
- What is the meaning of equivalence in this case?



c.f.: parallel resistors



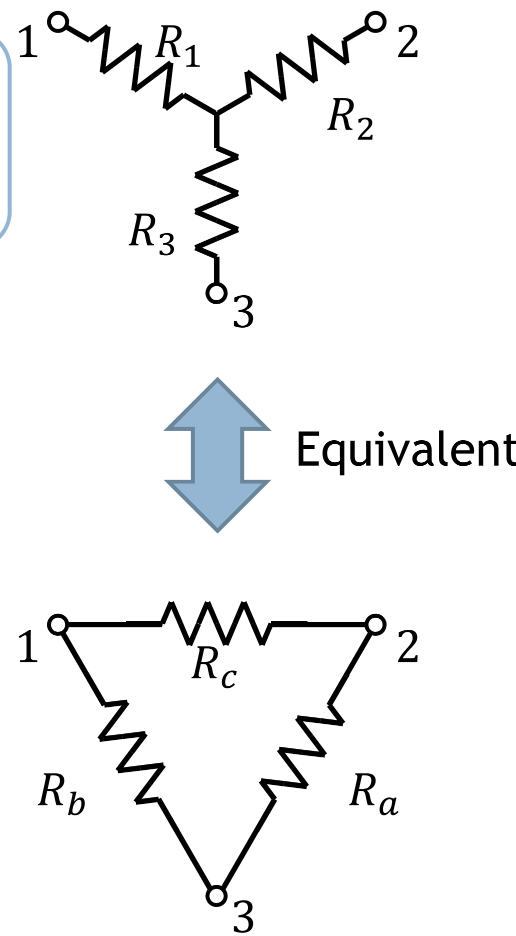
Equivalence:
For any v_{12}, the resulting current is the same.
(For any current i_1, the resulting voltage drop is the same.)

Wye-Delta transformation

- Equivalence for a three-node network:
  For any combination of v_1, v_2, and v_3, the resulting currents i_1, i_2, and i_3 are the same.
- Since this is a linear system, the relationship between voltages and currents can be expressed as the following:

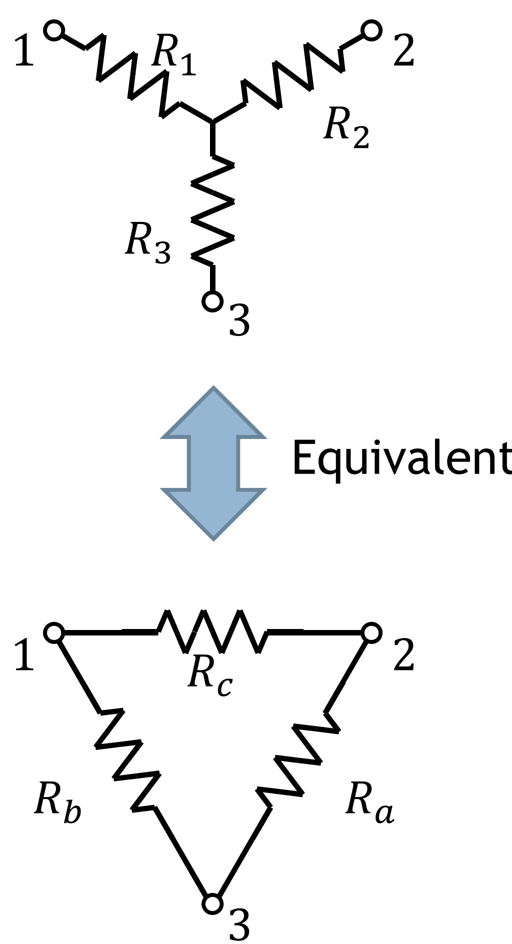
$$\begin{bmatrix} i_1 \\ i_2 \\ i_3 \end{bmatrix} = \begin{bmatrix} G_{11} & G_{12} & G_{13} \\ G_{21} & G_{22} & G_{23} \\ G_{31} & G_{32} & G_{33} \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix}$$

- Thus, equivalence means the same G matrix



Wye-Delta transformation

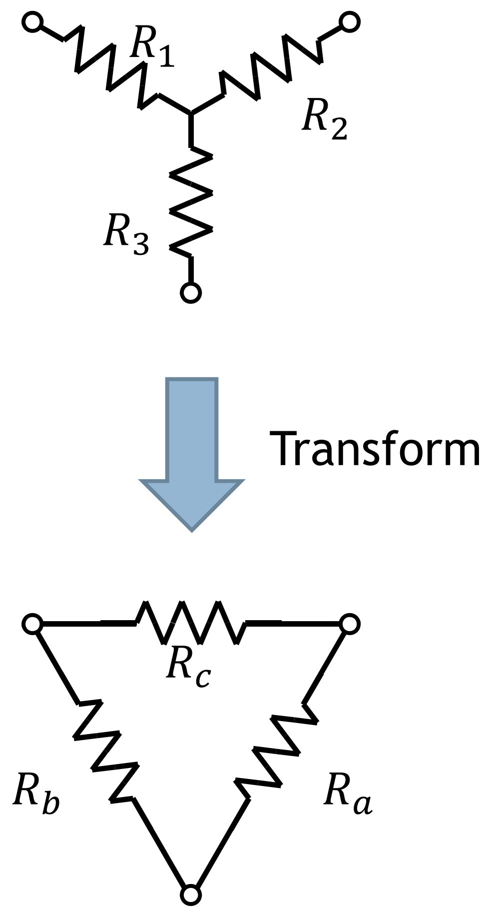
- $$\begin{bmatrix} i_1 \\ i_2 \\ i_3 \end{bmatrix} = \begin{bmatrix} G_{11} & G_{12} & G_{13} \\ G_{21} & G_{22} & G_{23} \\ G_{31} & G_{32} & G_{33} \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix}$$
- To confirm that the G matrix is the same, we only need to check the currents for three independent (v_1, v_2, v_3) combinations.
- For example, if the currents are the same for
  $$v = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \text{ and } \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \text{ and } \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix},$$
  then the currents are the same for any *v*.



Wye-to-Delta transformation

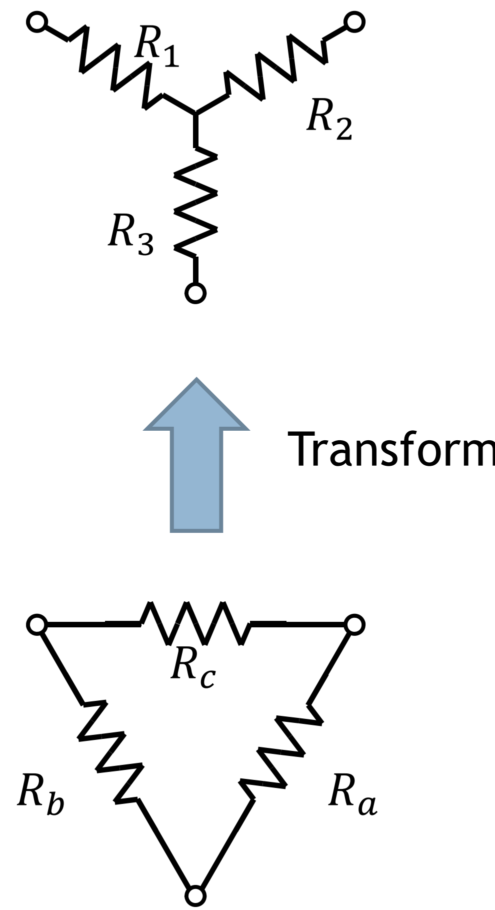
- Removes one node
  (What happens to n-b+l = 1?)
- $R_a = \frac{R_1R_2+R_2R_3+R_3R_1}{R_1} = R_2 + R_3 + \frac{R_2R_3}{R_1}$
- $R_b = \frac{R_1R_2+R_2R_3+R_3R_1}{R_2} = R_3 + R_1 + \frac{R_3R_1}{R_2}$
- $R_c = \frac{R_1R_2+R_2R_3+R_3R_1}{R_3} = R_1 + R_2 + \frac{R_1R_2}{R_3}$
- This can be easily checked if we look at

$$v = \begin{bmatrix}1\\1\\0\end{bmatrix} \text{and} \begin{bmatrix}0\\1\\1\end{bmatrix} \text{and} \begin{bmatrix}1\\0\\1\end{bmatrix}$$

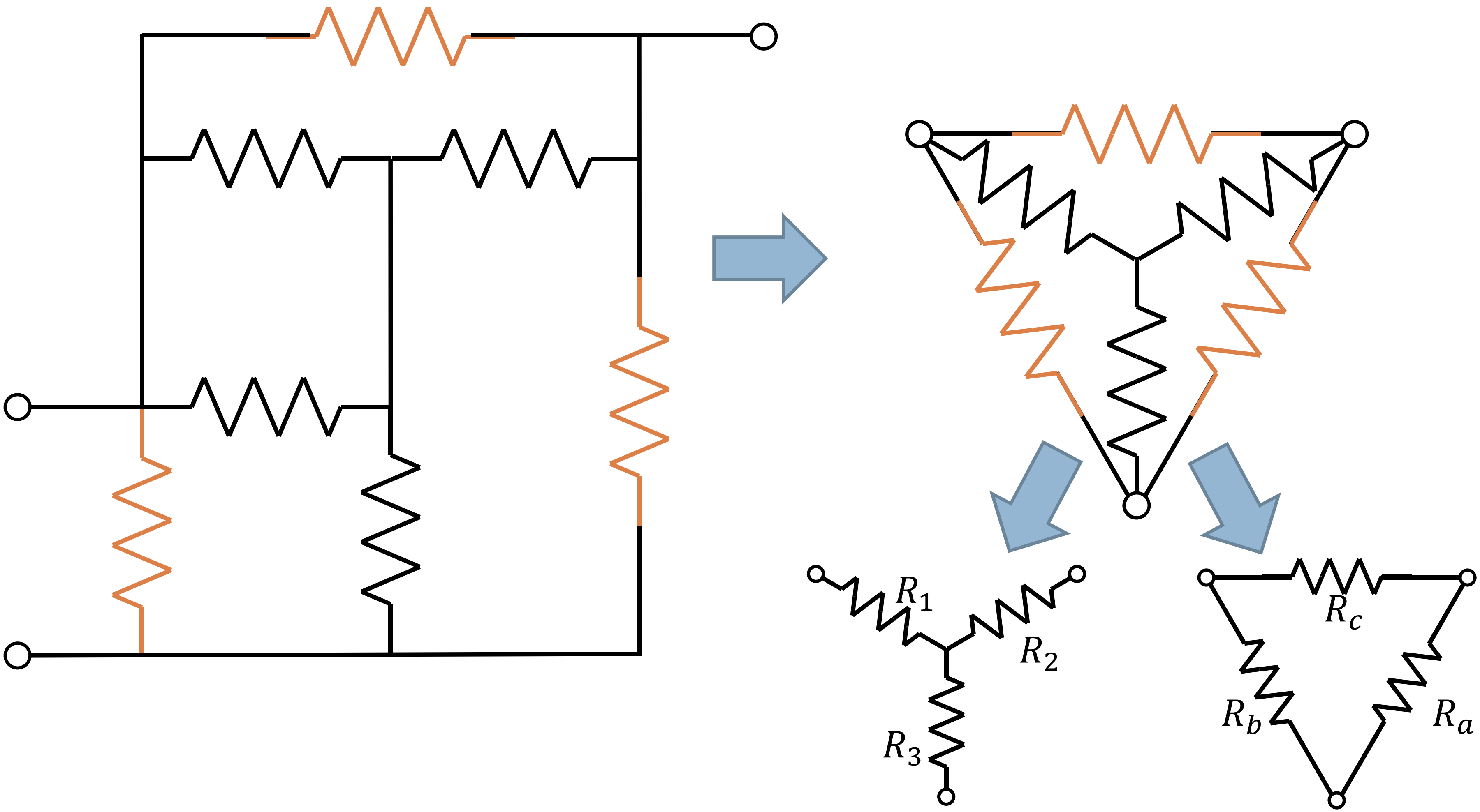


Delta-to-Wye transformation

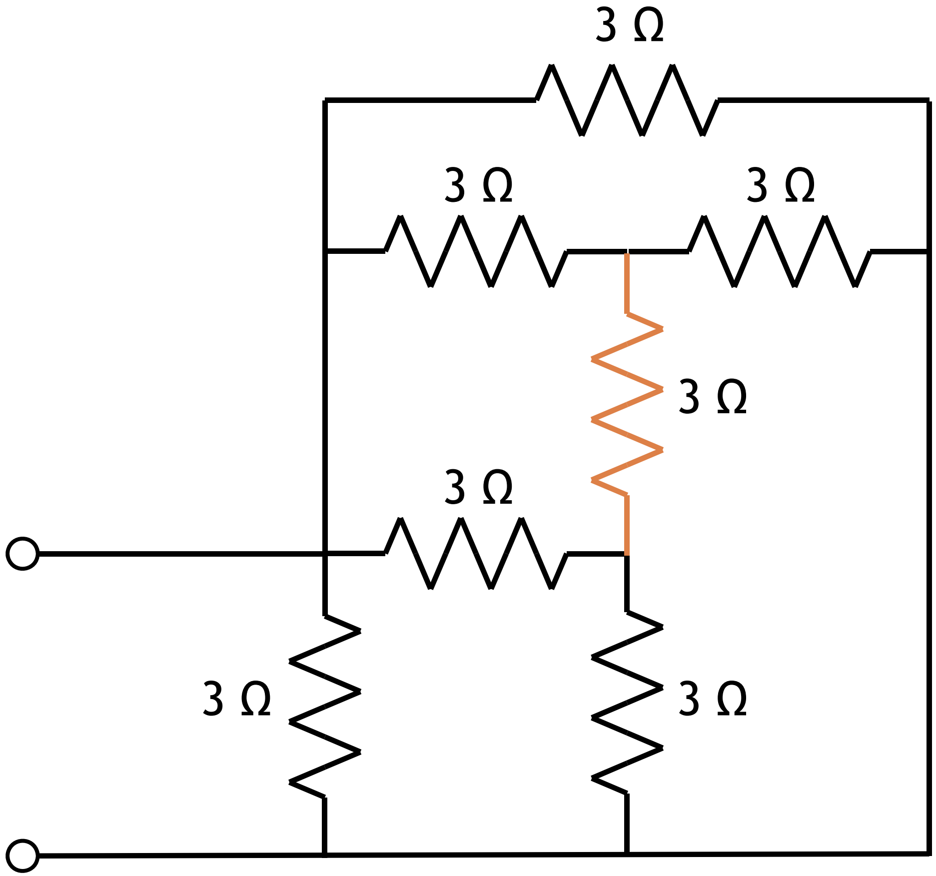
- Adds one node
  (Why adding one node can help simplifying?)
- $R_1 = \frac{R_b R_c}{R_a + R_b + R_c}$
- $R_2 = \frac{R_c R_a}{R_a + R_b + R_c}$
- $R_3 = \frac{R_a R_b}{R_a + R_b + R_c}$
- This can be easily checked if we look at
  $v = \begin{bmatrix} 1 \\ v_2 \\ 0 \end{bmatrix}$ that gives $i_2 = 0$ (open circuit)



Three-node resistor network example



Quiz



How many Delta networks are there?

Methods of Analysis

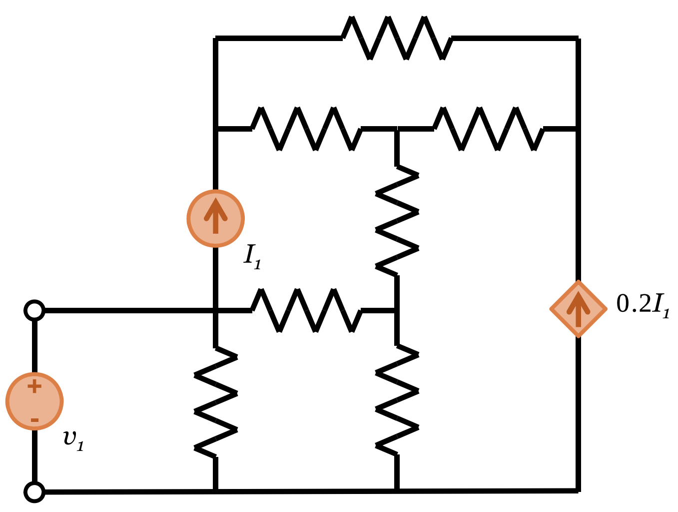
Nodal Analysis (Find v_n using KCL)

Mesh Analysis (Find i_n using KVL)

Circuit analysis

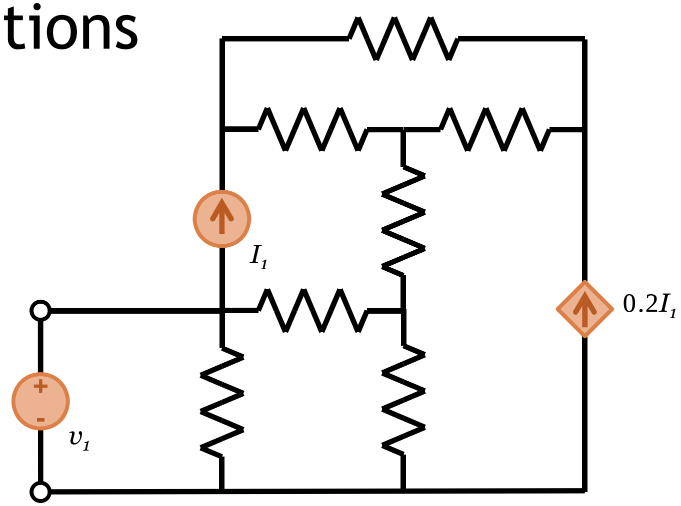
- What does it mean to analyze a circuit?
  - Find an equivalent resistance (impedance) ← Can be done with simplification techniques
  - Find voltages of each node
  - Find currents in each branch

Nodal analysis, mesh analysis



Nodal analysis

- Procedure
  1. Identify nodes
  2. Assign a voltage variable to each node except one node (reference node)
  3. For each non-reference node, apply KCL to get an equation
  4. Solve (n-1) simultaneous equations



Nodal analysis

- Procedure 2: Assign voltage variables