

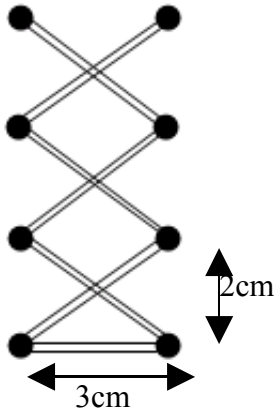
Name \_\_\_\_\_

## Pythagorean Shoe Laces

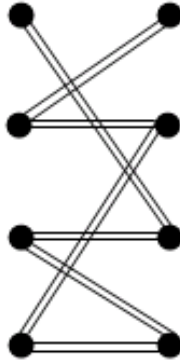
- Have you ever tried to tie your shoelaces and found they were too short?
- Is there a way of re-threading your laces to save some length for tying?

There are 3 different common methods for threading shoelaces:

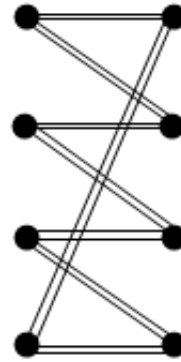
AMERICAN



EUROPEAN



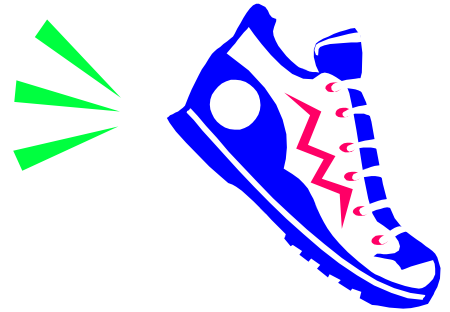
SHOE SHOP



If a shoe has 4 pairs of holes, with the opening between the holes 3cm wide, and the height between each pair of holes 2cm, we can work out how long the laces need to be for the shoe.

### American method

1. Work out the length of each diagonal thread using Pythagoras.
2. Work out how many diagonals there are.
3. Add on the horizontal length to find the total length of shoe lace needed.
4. Can you write this as one big sum (no workings out along the way)??



### European method

1. This time there are two different diagonals. Work out the length of each one using Pythagoras.
2. Write down how many of each type there are.
3. Add on any horizontal lengths to find the total length.
4. Can you write this as one big sum?

## Shoe Shop method

1. Again there are two different diagonals. Work out the length of each one using Pythagoras.
2. Write down how many of each type there are.
3. Add on any horizontal lengths to find the total length.
4. Can you write this as one big sum?

**Conclusion: Which method uses the least lace? .....**  
**Which method uses the most lace? .....**

Note: If you weren't able to do the 'one big sum' parts you need to ask for help now...

Not every shoe has four pairs of holes, there is not always 3cm between the holes and the pairs may not be 2cm apart. Will the answer still be the same? We need to do some more maths to find out...

We need some algebra:      Call the number of pairs of holes 'n' (n=4 in our example)  
   Call the distance across between the holes 'd' (d=3 in our example)  
   Call the gap between the rows of holes 'g' (g=2 in our example)

## American method

1. Write out the 'one big sum' again, but put in 'd' and 'g' instead of the 3cm and 2cm parts.
2. The other number in the sum is the number of diagonals. We need to work out the link between the number of diagonals and the number of pairs of holes (n). Set up a table in EXCEL like the one below:

Number of pairs of holes (n)	Number of Diagonals
2	
3	
4	
5	
6	

Fill in the second column. What is the pattern?

3. Write down the formula for the number of diagonals in terms of n.
4. Now write out the 'one big sum' for the last time, but this time replace the number of diagonals with the formula you worked out above.

# CONGRATULATIONS!!!!!!

If you have received this sheet then you must have worked out that the length of shoelace for ANY shoe for the American method is:

$$g + 2(n-1)\sqrt{d^2 + g^2}$$

We can use this to make a spreadsheet do some hard work for us!

In a new spreadsheet set up a table like this:

B3		=		=B2		
	A	B	C	D	E	F
1	<b>n</b>	<b>d</b>	<b>g</b>	<b>American</b>	<b>European</b>	<b>Shoe Shop</b>
2	2	3	2			
3	3	3	2			
4	4					

Notice the formula bar says B3 '=B2'. Make the 'n' column go down to at least 10, and copy the '=B2' all the way down 'd'. For 'g', type '2' into cell C2, and the '=C2' into cell C3 and copy this all the way down.

In cell D2 we need to type our big formula, using A2 for 'n', B2 for 'd' and C2 for 'g'. Always start a formula with '='.

We also need to remember to:

- Use '\*' instead of 'x' for 'multiply'
- Use '^2' to mean 'squared'
- Use 'sqrt' to mean 'square root'

When you think you have done it, copy the formula down the rest of the column. If you are right, the answer when n=4 will match your answer on the first sheet.

## Challenge

A. Work out the formulas for the European and the Shoe Shop methods, starting with your 'one big sum' answers and turning them into formulas with 'd', 'g' and 'n'? (Use the back of this page for workings.)

B. Type these formulas into cells E2 and F2 respectively and copy them down to work out which method is best when d = 3 and g = 2.

C. Now try changing d and g (you only need to change the top cell of the row) and see if it gives you a different answer. Write about what you found out:

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