The role of examples in Cognitive Apprenticeship

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Abstract

In this paper we demonstrate some examples of mathematics of everyday practice. In other words, we show examples of mathematics that if introduced in the teaching classroom, then they may be proved to be important instructional tools for the enhancement of teaching-learning process. The examples presented below, give a meaning to the purpose of learning mathematics, represent interesting problems, and attract students' interest to the learning of mathematics. One important thing is their instructional transformation and their transfer in the classroom, regarding the learning theory, and the context, in which, this theory should be used. In this paper we publish the results of a related research, where we employed the Cognitive Apprenticeship model as a learning theory.

Introduction

Recent views consider learning as an indispensable part of everyday life. Problem solving and learning are fundamental procedures of "embodied" or "situated" learning. In other words, learning is not detached from the world of action; rather it exists in complex social contexts that are constituted by people, actions, and situations. It was the idea of sociocultural theory of Vygotsky (1993; 1998; 2003) that led Lave and Wenger (1991) to formulate the model of situated learning, which posits that learning involves a participation in communities of practice or else known as "learning communities". Brown, Collins, and Duguid (1989) mention that, research on learning put into doubt the traditional instructional methods used in the classrooms. Those traditional instructional methods often split knowledge into abstract, decontexturalized concepts, theoretically independent from the situations, in which the knowledge is gained and used. They also believe that the activity, in which the knowledge is gained and used, represents a united and indispensable part of what is learnt, and also they believe that situations generate knowledge through activities. Thus, they suggest that learning and knowledge are deeply situated (Brown et al., 1989).

cognitive apprenticeship

Over the centuries, apprenticeships have proved to be an effective form of education. In an apprenticeship, the basic participating roles are that of the learner, of the master, and that of the learning activity. By working alongside a master and perhaps other apprentices, young people have learned many skills, trades, and crafts. The apprenticeship system often involves a group of novices, students, who serve as resources for each other in exploring the new domain and aiding and challenging one another. The expert or teacher is relatively more skilled than the novices, with a broader vision of the important features of the activity.

Since early 90's, educational researchers, and especially those that study educational technology, have proposed several theoretical frameworks, in an attempt to guide the design and the use of learning environments. One of these framework is the theoretical construct of "cognitive apprenticeship". The term was firstly coined and formulated by Collins, Brown, and Newman (1989: p.453), who mentioned:

> "…we propose in alternative model of instruction that is accessible within the framework of the typical American classroom. It is a model of instruction that goes back to apprenticeship but incorporates elements of schooling. We call this model "cognitive apprenticeship" (Collins, Brown, & Newman, 1989)."

Collins, Brown and Duguid (1989), in a project about the evolution of the method of cognitive apprenticeship, found that many teaching practices suppose that conceptual knowledge can be derived from situations, wherein the knowledge is gained and used. Brown et.al. (1989) notice that when authentic situations, that are similar to those, wherein knowledge is applied, are created during a learning process, then they affect the relationship between the learning situation and the final working situation, thus facilitating learning. Brown et.al. (1989) conclude, based on recent research relating to the field of knowledge, that knowledge is "situated", partially being a result of the interaction with the activity, the context, the culture wherein it is gained and finally used.

The model of situated learning may well provide remarkable ideas about new pedagogical methods and practices. Specifically, learning can be realized through experiential learning (Anderson et al., 1996), and knowledge is not any more a privilege of few people rather it can be easily gained through discussion in learning communities. A problem that seems to arise is how teacher can create authentic situations, in which the students will be engaged. This problem seems to be successfully faced by technology.

Collins, Brown, and Newman (1989), Collins, Brown, and Holum (1991), and Collins (1991) argued that effective learning environments could be characterized through 18 features belonging to four broad dimensions (or building blocks), namely content, methods, sequencing, and sociology of teaching (see Table 1). Many parts of this model are not new, but together they define an effective learning situation, with very different classrooms and roles for teachers and students.
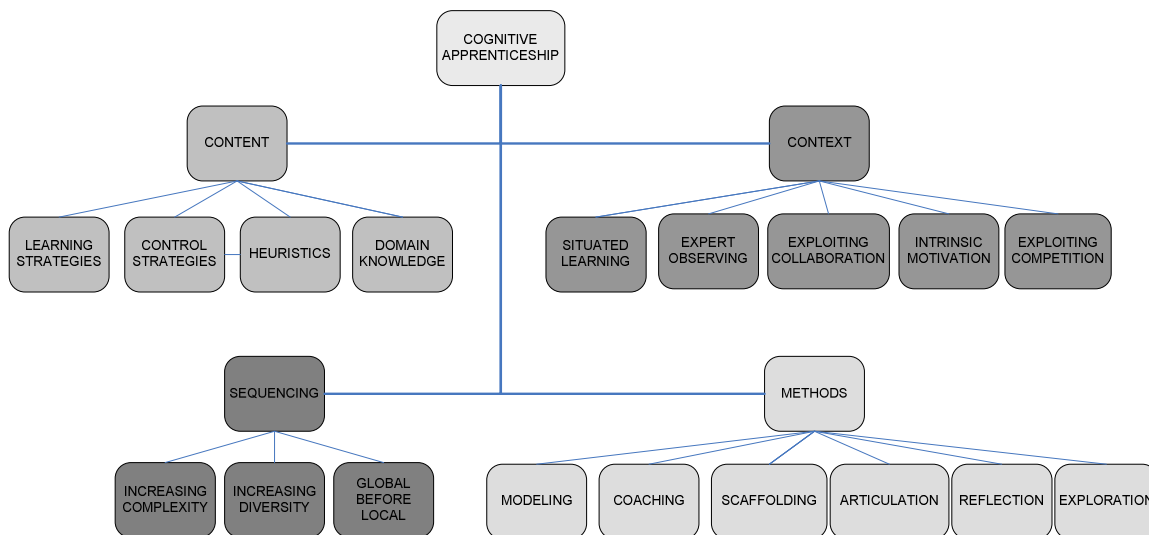
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Figure 1

The teaching methods of cognitive apprenticeship (modeling, coaching, scaffolding, articulation, reflection, and exploration) create the appropriate conditions for the application of active learning.

Teaching Methods	Aim
• Modeling • Coaching • Scaffolding	To help students gain an integral set of cognitive skills by observation and supported practice, while teacher fades out, assigning the student to complete the activity.
• Articulation • Reflection	To i.e. to explain and reason their actions on what they do or to compare what they know with what other know, so as to clearly formulate their ideas, and justify their thoughts.
• Exploration	To encourage students' independence, problem formulation by them, and transfer of knowledge from teacher to them

Table 1 Cognitive Apprenticeship Teaching Methods

The six teaching methods, in turn, break down into 3 groups. The first group consists of modeling, coaching, and scaffolding. The second group consists of articulation, and reflection. The third group consists of exploration.

One critical issue in cognitive apprenticeship model is that the activities require the existence of an authentic context. Authentic context has been widely discussed in the literature and numerous research papers and written references that evaluate its contribution in a learning environment exist which evaluate it.

Jonassen (1991) suggested that situated learning is an effective instructional paradigm for advanced knowledge acquisition. "Environment" plays an important role in situated learning. Rogoff (1984; p.2) defined "environment", in the context of learning environments, as "the physical and conceptual structure of the problem and also defined the aim of the activity and the

social context, inwhich the activity is embodied". McLellan (1994) stressed that the context, in learning environments, may be provided either by real problems, or by a realistic substitute of the working environment, e.g. video, or multimedia software.

THE role of Technology

Computers, as cognitive tools (Jonassen, 1996), provide substantial help to students through the six teaching methods of cognitive apprenticeship (Collins, 1991, De Corte, 1990; De Bruijn, 1993b; Wilson & Cole, 1991) because the context-specific learning is ensured by the computer that simulates real-world situations. Thereby, students eventually become aware of the conditions, in which the knowledge will be applied, discover which knowledge must apply each time, its applications, and therefore can use it more easily in similar situations. Computer offers dynamic tools, which allow pupils to explore quicker the hypotheses and the possible solutions of a problem.

Computers' assistance in Methods of Cognitive Apprenticeship

In *modeling*, computers can model processes bringing to light the invisible by showing the strategies that experts use to solve problems. Here, multimedia play a central role in the whole process.

In *coaching*, computers have the ability to patiently monitor students, and to provide hints and just-in-time help, when this is necessary. This kind of personal monitoring is not practically possible in most classrooms.

Reflection is facilitated by computers, since it provides deductive repetitions and pinpoints the main characteristics of a student's effort. In this way, students analyze their performance from different views and may compare their solutions to those of experts or to those of other classmates.

In *articulation*, computers make students build their own theories or ideas in a virtual reality environment, and even more computers may provide tools and context that permit students to express their ideas to the rest part of the classroom.

In *exploration*, computers challenge students to develop conjectures about the methods, and the strategies they use in problem solving, since they give them the opportunity to directly observe their results in the simulated situation.

THE ROLE OF EXAMPLES IN COGNITIVE APPRENTICESHIP

The use of examples, no matter which is the instructional model the teacher follows, is a key point of the instruction, because the example functions as a tool that aids the presentation of the subject matter, helps the teacher to clarify the concepts, serves as the starting point for the students to develop conjectures, and also as a point for the rejection of those conjectures, when it works as a counterexample. Generally, the example helps in many ways towards the enhancement of understanding. Thereby, examples often construct, concretize, other times over-simplify, enrich, or establish knowledge. Such an example like the one that Guido Grandi (1671-1742) noticed can help the students to bring out that the partial sums do not converge to a fixed value, and thus the series diverges. In 1703, Guido Grandi noticed that from the infinite series:

$$1-1+1-1+1-1+1-1+1-1+1-1+...$$

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it is possible to obtain 0 or 1. Notice that:

$$(1-1)+(1-1)+(1-1)+(1-1)+... = 0+0+0+0+... = 0$$

$$1+(-1+1)+(-1+1)+(-1+1)+... = 1+0+0+0+... = 1$$

The sum series of course, merely shows that the partial sums do not converge to a fixed value, and thus the series diverges (though not to infinity).

An example taken from the history of mathematics, such as the paradox of Zenon of Elea can help teacher to introduce new concepts and, in this case, to introduce the concept of endless geometric progression. The Greek philosopher Zeno of Elea, who lived in the fifth century B.C., purported to show that motion is impossible: in order for a runner to go from a point A to another B in the following way: the runner must first cover half the distance between point A, B. Then, he covers half the remaining distance, then half of that, and so on, ad infinitum (Figure 1).

Since this involves an infinite number of steps, Zeno argued, the runner will never reach his destination. It is easy to formulate Zeno's paradox in modern terms. If we suppose that the distance from A to B be 1; by first covering half this distance, then half of what remains, and so on, the runner will cover a total distance given by the sum:
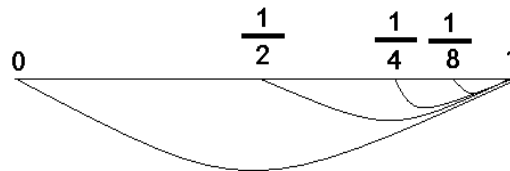

Figure 2

$$\frac{1}{2}+\frac{1}{4}+\frac{1}{8}+...$$

Due to the fact that this sum represents an endless geometric progression, with the common ratio ½, the teacher can stimulate students' interest.

Nowadays, with the help of ICT (Information and Communication Technologies), teacher has the potential to maximize the didactical contribution of the example in his/her instruction. In this way, the example may be utilized by the teacher in a more dynamic way, e.g. as a tool that is more picturesque with the aid of computer, and thus more effective to inforce student's cognitive development. Teacher may use the example to create conditions of motivation, of questioning, of cognitive conflicts, and reflection, in order to help student construct the knowledge in his/her own. If we take into account, what Comenius mentions in "Didactica Magna" (Comenius, 1592-1670), everything, if possible, should be presented to students; to all senses; what can be observed to the vision; what can be heared to the hearing; what can be smelled to the smell; what can be tasted to the taste, and what can be touched to the touch. Generally, if something can be seized in other senses, must be simultaneously presented to these different senses. Computers seem to satisfy this claim.
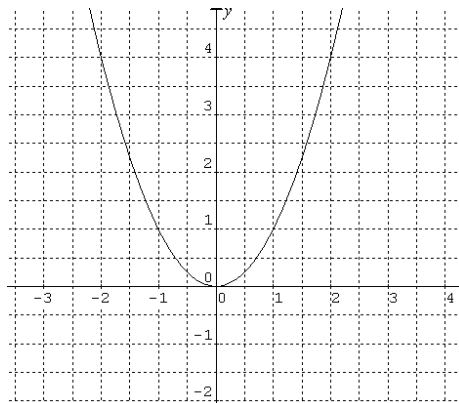

Figure 3

However, often arise some questions, like: Can the examples on computer screen substitute proofs when confirmating or disproofing mathemati-
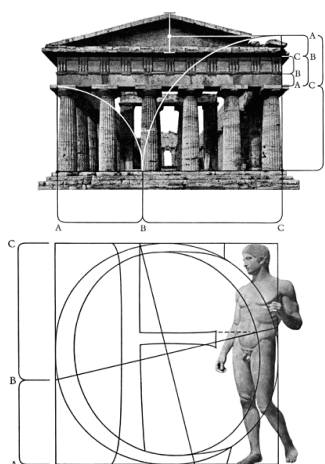
cal propositions, given that new technology convinces the student for good? Particularly, technology convinces student, mainly through multiple, and dynamic representations, for the validity of a statement. A student often is so sure about the truth of a statement that accepts it without considering necessary to prove it. For example when the student looks at the graph of $y=x^2$ (Figure-3) he feels sure about the fact that the graph from the origin of coordinates so he thinks that it is unnecessary to prove it. Furthermore, simulations put student in the center of the teaching-learning process.

Unlike traditional methods, cognitive apprenticeship model doesn't split knowledge into abstract and decontexturalized concepts from real-world situations. Rather, it unifies the knowledge with the context and the situations, in which the knowledge is gained, and used. Moreover the computer enables simulation of real-world situations in a dynamic way. Thus, the examples in cognitive apprenticeship model can be more explanatory and meaningful. Under these conditions, we can use cognitive apprenticeship, computer, and appropriate example to engage student in an effective learning situation.
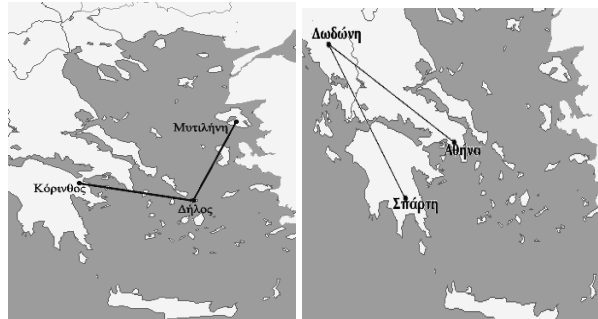
Examples

An example can become, with the support of computer in a cognitive apprenticeship context, a valuable assistant toward the understanding of the subject matter and the engagement students with actions. According to Mariotti (2003) "Dynamic Geometry Software, offer a powerful environment incorporating the semantic domain of space and time, where the key notions of geometry can be grounded. This general idea can be interpreted in a Vygotskian perspective, according to the notion of semiotic mediation". Balacheff (2001) asserts that this passage from computer's monitor to mathematics represents a modeling process, and maintains that it is necessary for the teachers to be acquainted with this process. Indicatively, we mention some examples below. Of course, a teacher can find many more of them by himself on every subject matter, for example on the concept of vector (Gagatsis, & Demetriadou, 2001).

1. The front of Parthenon represents an appropriate example that may well be used to teach the "golden section". Furthermore, the ratio of two successive numbers that belong to Fibonacci sequence tends to be equal with the number of the golden section. Moreover, the proportions of human body follow the rule of golden section, too. Students, who use the computer, can work on the above examples in their context.

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2. The concept of equal distances that lead to the concept of isosceles triangle through the example of the distance between three cities.


3. The estimation of the length of a ladder or the construction of a housetop are two appropriate examples to use when teaching Pythagorean Theorem. ICT provide substantial help to students because computer simulates real-world situations.
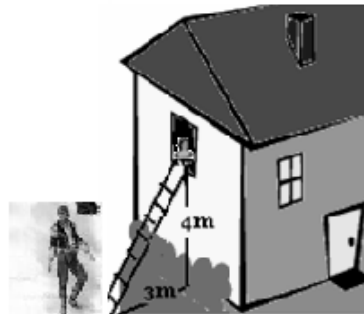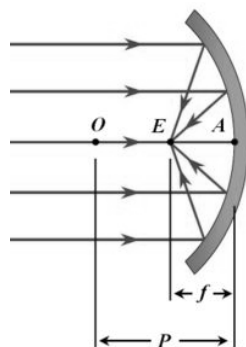

4. The student – computer interaction before the teaching of the concept of slope acts as an advanced organizer, according to Ausubelt(1960).
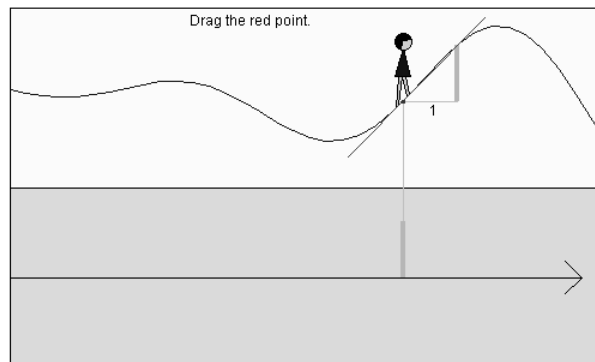
5. The orbit of a comet or the shape of a car's headlights is an appropriate example to teach parabola.
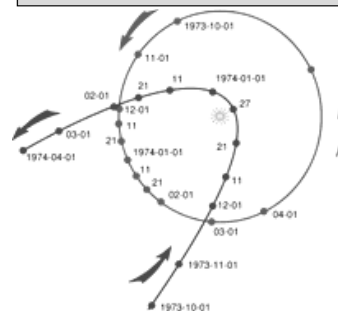
RESEARCH

Our main research goal was to make obvious that computer-supported instruction in a cognitive apprenticeship context with the simultaneous use of examples provides more significant learning outcomes than a traditional instruction does. We intentionally selected to teach the concept of perpendicular bisector from Euclidean geometry. We taught the control group the perpendicular bisector employing a traditional instructional method. On the other hand, the experimental group was taught the perpendicular bisector employing the cognitive apprenticeship model that involves the use of numerous examples to make explicit the knowledge of expert to other students. According to the research results, students in experimental group had significantly better performance than the students in control group.

METHOD
Sample

The sample of the research consisted of two first-year classes (16 year-old students) in at a public high-school in Peristeri, Athens. The first group (experimental) involved 20 students (11 males and 9 females) and the second group (control) involved 23 students (11 males and 12 females).

Instruments

Pre-tests and post-test were developed in order to collect data on each group's performance. Initially, we administered a pre-test that contained three problems. After the instructions had been completed, we administered a post-test that contained three problems of equal level of difficulty with that of the pre-test, so as to keep stable the tests, and focus solely on the effect the different instructional model would have on students' performance.

We also used several software tools during our research. We asked the students in the experimental group to use the following software tools: Geometer's Sketchpad, Macromedia Flash Player, Java applets, web-browser and graph applications. We also handed a corresponding activity sheet to the teachers.

Procedure

The traditional method i.e. using the chalkboard and without using computer technologies was used in our instruction in the control group. On the contrary, in the experimental group, we employed the cognitive apprenticeship instructional model that involves the presentation of many examples so as to investigate the effect of examples on students' understanding. The courses, i.e. a) perpendicular bisector b) Pythagorean Theorem and c) areas of plain figures,

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were taught in the control group by an experienced teacher employing traditional instructional methods; in the experimental group by us employing the cognitive apprenticeship instructional model. The courses lasted 3 weeks. More analytically, a course took place two times per week, two hours per time for every topic in every group. A test, that contained three exercises, was given to the students before the beginning of instructions (pre-test). Each exercise was scored between 0-6 with 6 indicating an excellent answer. We mention that the notions that were taught were not completely unknown to the students, because they already had been taught them earlier in high school. A test, that contained three exercises, was given to the students after the instructions (post-test). Each exercise was scored in the same way as pre-test i.e. between 0-6 with 6 indicating an excellent answer.

RESULTS

The pre-test was given to the students to check whether the background knowledge of students before the use of cognitive apprenticeship were different. The comparison of students' mean scores in pre-test i.e. we run an independent samples t-test in both groups, control and experimental, revealed that the two groups had not statistically significant differences in their performance (t (41) = .32, $p>.05$).

A 2x2 mixed repeated measures ANOVA with one within-subjects factor (time (time1 =pre, time2= post measures)) and one between-subjects factor (group (control versus experimental group)) was conducted to evaluate the effect of group and time on students' performance. The ANOVA yielded a significant time main effect ($F_{(1, 41)}=137.82$, $p<.001$)) and a significant group main effect ($F_{(1, 41)}=19.24$, $p<.001$). The interaction between the pre-post measures and the group was found to be statistically significant ($F_{(1, 41)} =85.63$, $p<.001$).

Also, a paired samples t-test (between pre-test and post-test scores of students in experimental and control group) was conducted to follow up the significant interaction. Table 1 presents pre- and post measures scores and change scores for the experimental and control groups.

Table 1

	N	TIME1 MEAN (STD)	TIME2 MEAN (STD)	MEAN DIF. (STD)	P VALUE
Experimental group	20	11.26 (2.15)	14.35 (2.56)	-3.09 (1.48)	*p<.001*
Control group	23	10.51 (2.12)	11.13 (2.19)	-.62 (1.27)	*p>.05*

The results (Table 1) revealed that the children in the experimental group made a significant improvement in their performance (t (19) = -10.46, $p<.001$), in comparison with the students in control group, who didn't showed a statistically significant difference in their performance between pre-test and post-test (t (22)= -1.89, $p>.05$). In addition the same ANOVA was conducted to evaluate the effect of sex (boys versus girls) and time (pre - post measures) on students' performance. The ANOVA yielded a significant pre-post main effect ($F_{(1, 41)} =78.21$, $p<.001$) and a non significant sex main effect ($F_{(1, 41)} =.09$, $p>.05$). The interaction between the pre-post measures and the sex was found to be non significant ($F_{(1, 41)} =.23$, $p>.05$).

CONCLUSION

We propose, based on the factors mentioned above, that teachers should choose suitable examples so as to engage students into Computer Supported Collaborative Learning environments in the teaching of the methods of cognitive apprenticeship. The selection of cognitive apprenticeship can help the teacher to enhance his/her instruction through use of appropriate examples. The selection of cognitive apprenticeship can help the student through examples that come to life with the aid of direct manipulation on computer's screen. Cognitive Apprenticeship can also help student to better understand the concepts, given that student apprehend the real world through his/her daily experiences.

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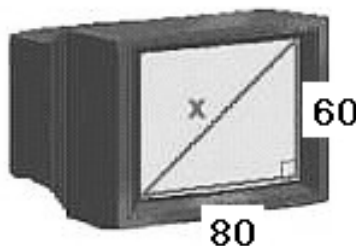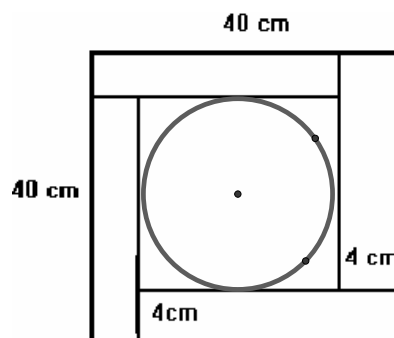
APPENDIX

Pre-test problems

1. *Where should the bus station be built, so that the residents of a village, and the workers of a factory that live on both sides of the road to be at the shortest possible distance from the bus station? Justify your answer.*
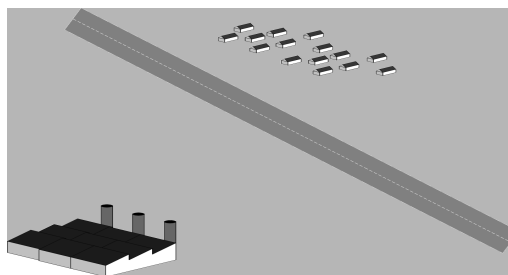
2. *What is the diagonal length of a TV screen whose dimensions are 80 x 60 cm?*

3. *Four strips of paneling 40 cm long and 4cm wide are arranged to form a square. What is the area of the inscribed circle of inner square (in cm^2)?*

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Post-test problems

1. *Where should the bus station be built, so that the residents of the two villages, that are located at both sides of the road, be at the shortest possible distance from the bus station? Justify your answer.*
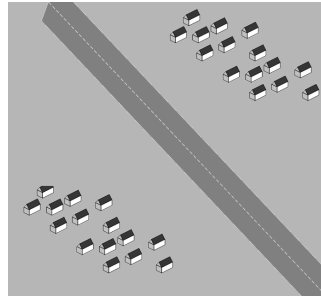
2. *Which is the length of the ladder that Romeo needs to put against the wall so as to reach the open window of the house, if in front of the wall there is a parterre, and Romeo must constrainedly put the ladder at least 3,6m away from the wall?*

    4,8m

    3,6m
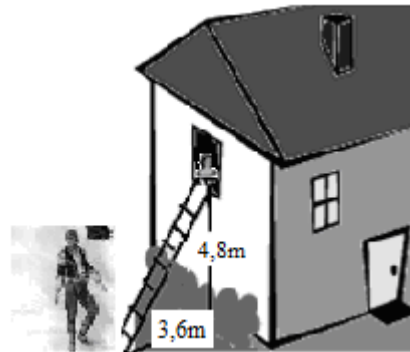
3. *The Gillis's house has a pool with the shape as shown. They want to make a cover for it for the harsh winter. How much to the nearest cent are they going to have to spend on material if it costs $5.00/m^2.*

    6m

    4m

    9m