Using Learning Objects to teach Euclidean Geometry

Emm. Nikoloudakis(Ph.D., M.Ed., M.Sc.)[1], G. Dimakos[2]

[1]Headmaster of 7th Lyceum of Peristeri
enikolou@otenet.gr
[2]Associate Professor
Faculty of Primary Education
University of Athens
gdimakos@primedu.uoa.gr

Abstract

Learning objects have gained a leading role in a modern ICT-enriched learning context, due to the goals and features that seem to embody, despite existing difficulties. The technology of these learning objects is a driver for change and important technological innovations that can lead to significant review of concepts, processes and functions held in a certain way so far in education. In this article, we aim to explain to mathematics teachers, who claim that they are not familiarized with computer use, and therefore do not deploy it in their teaching, that they can teach with the use of ICT even though they are not computer experts. The only thing teachers need to be aware of is a frame of core skills of the World Wide Web, because it offers teachers a great variety of Learning Objects, so as to plan and organize their computer supported teaching.

Keywords: Learning Objects, Model of p-m combinations, Structured Form Worksheet

1. Introduction

Teaching Euclidean Geometry represents one of the most crucial subjects in all education levels. Nevertheless, students face great difficulties in the understanding of concepts and processes of geometry. As a result, numerous mathematics education researchers have focused their research interest on the teaching of geometry for several years [Papert (1980)];[De Villiers, (1997); (1999)]; [Tall (1993)]; [Zaranis et. al (2002)]; [Mariotti (2003)]; [Koleza (2000)]. What is worth mentioning is that, nowadays modern educational media, such as the existence of school computer

laboratories, the variety of educational software, and the easiest access of students in computers, due to their lower cost of adoption, improve teaching situations. The systematic use of special educational software helps passive learning to become more active [Kritikos (2004)], while [Kalavassis (1997)] notes that the skills required in a technological environment are in total agreement with the teaching suggestions by learning theories and epistemology, in the way those are formulated by the Didactics of Mathematics. Although, the interaction between students and software does not represent a substitute of some situations such as the overcoming of obstacles or the construction of a proof, it constitutes a unique blend of technology and careful thought, which outlines a path that ultimately leads to new situations such as the overcoming of obstacles [Nikoloudakis et.al (2008)], or the construction of a proof [Kritikos et.al (2006)]. In other words, technology is an important lever of change and technological innovations that can lead to significant revisions of concepts, processes and functions, particularly in education, held in a certain manner to date. Nowadays, internet can play a special role in the process of teaching-learning. Students and teachers have the opportunity to rapidly communicate from any place on earth with each other or with practitioners and experts on a negotiating problem. Proper use of technology expands the environment and horizons of the classroom [Vlamos et.al (2003)]. Apart from adequate communication, digital repositories that are accessible via the Internet may supply teacher with a rich repertoire of learning objects, which enable the teacher to use them in teaching. This scenario will be examined below.

2. Learning Objects and Instruction

2.1 Learning Objects

Although an effort to train teachers in modern technologies of information and communication has already been made in Greece, through specially designed training programs, the concept of "learning object" still represents a rather unknown term for the majority of teachers' community. Nevertheless, it is almost sure that those practicioners, who systematically use computers, have come across various learning objects that impressed them and probably used them without necessarily knowing their exact terminology. For this, following we shed some light on the term "learning objects".

According to [Wiley (2000)] a learning object (LO) is a resource, usually digital and web-based, that can be used and re-used to support learning. Examples of smaller reusable digital resources include digital images or photos, live data feeds (like stock tickers), live or prerecorded video or audio snippets, small bits of text, animations, and smaller web-delivered applications, like a Java calculator. Examples of larger reusable digital resources include entire web pages that combine text, images and other media or applications to deliver complete experiences, such as a complete instructional event. The exploitation of LO's is directly linked to the teaching-

learning process, and therefore has a strong relationship with the learning theories. [Wiley (2000)] also posited and presented three components of a successful learning object implementation: an instructional design theory, a learning object taxonomy, and "prescriptive linking material" that connects the instructional design theory to the taxonomy, providing guidance of the type "for this type of learning goal, use this type of learning object". In addition to providing a worked example of this process, [Wiley (2000)] also presented design guidelines for the five learning object types.

Behaviorism, cognitivism and constructivism are the dominant theoretical positions in the field of computer-supported and web-based learning, at whatever level the theory is being used for the development of interactive web-based educational material or e-learning systems [Jonassen (1991)]; [Atkins (1993)]; [Deubel (2003)]; The differences in the development and use of a LO from every learning theory, regarding its structure, the information flow, the activities flow, and the level of interaction between students and computer are self-evident. According to [Atkins (1993)] to develop web-based educational material based on the theory of Behaviorism, we must first break down the subject matter into small chunks and then deliver it in the form of rules and definitions.

The navigation through the material must follow a linear design or with few branches (either through conditions or not) and should give the opportunity to move between previous / next sections of the LO. The succession of the material is made from the simplest to the most complex. Based on their performance in a given test, students may repeat some parts of the material of the LO or omit some others. Cognitive learning design approaches require that the LO is designed in such a way that higher-level learning objectives are attained through a rich set of activities. Therefore, the designer of the LO should be aware of the fact that cognitive theories emphasize on the active role of participants (students) in the process of knowledge acquisition. As students are actively involved with the subject matter, there should appear teaching techniques like e.g collaborative activities, discovery learning activities, activities that control the acquired knowledge etc. [Deubel (2003)]. To meet the requirements of the constructivist paradigm, [Jonassen (1991)] suggests that learning environments are supposed to integrate activities and real-world scenarios, provide multiple representations of reality or of the subject matter, focus on knowledge construction, encourage students to think using their experiences, and support the collaborative construction of knowledge through activities, where cooperation between students
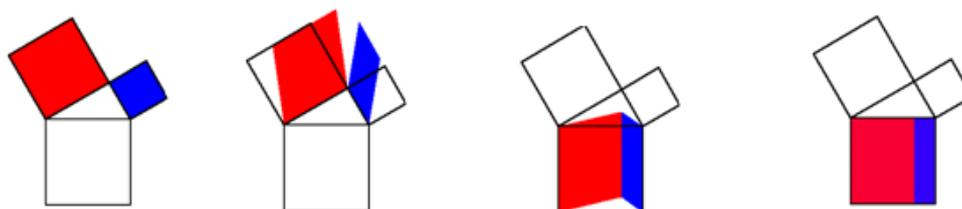

Figure 1

and adoption of group roles are required. For example, in Figure 1 we can see a flash animation that describes the validity of Pythagorean Theorem. What students can observe in this LO, is that the areas of the squares of vertical sides of a right triangle are equal to the area of the square of its hypotenuse.

Similarly, in the next flash animation (Figure 2) students can observe that the white space has the same area in both diagrams because the triangles do not chance size within the containing square. Therefore
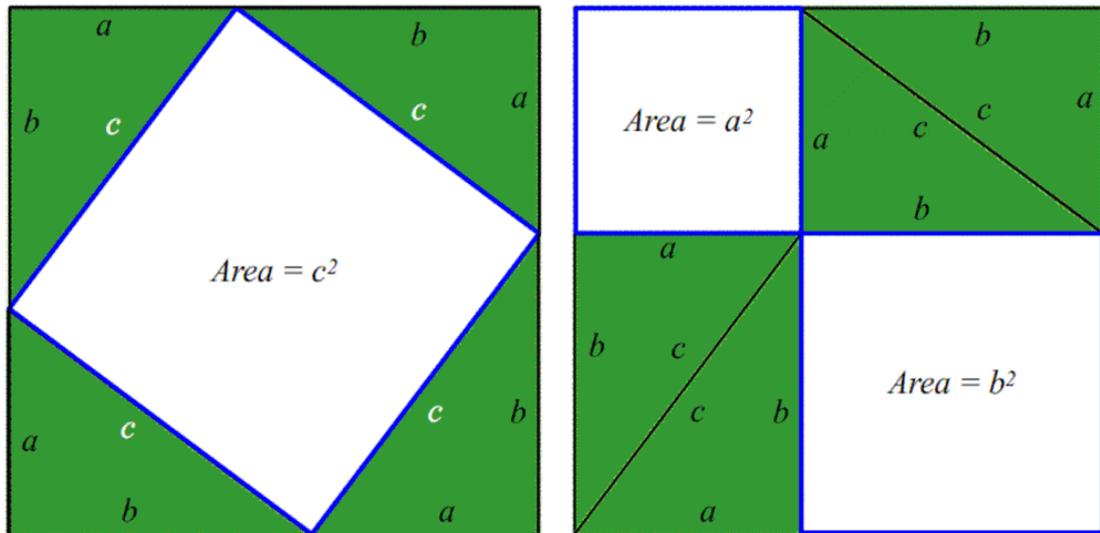
$$a^2 + b^2 = c^2$$

Figure 2

Every of this LO is a representation. In his research on the cognitive development of children, [Bruner (1966)] proposed three modes of representation:

- Enactive representation (action-based): it involves encoding action-based information and storing it in memory.
- Iconic representation (image-based): this is where information is stored visually in the form of images.
- Symbolic representation (language-based): this is where information is stored in the form of a code or symbol, such as language. This is the most adaptable form of representation, for actions & images have a fixed relation to that which they represent.

Bruner also posits that these three modes co-exist independently of child's age, and therefore every topic can be taught effectively to every child in a mentally fair way by communicating in each child's language. The environmental factors act and the experience, and therefore learning, that is treated as a process between student and the cultural and communicative elements of the environment. The above mentioned LO fits is fully compatible with Bruner's theory.

2.2 Learning Objects and Learning Theories

As mentioned above, the design of a LO is not random but it is based on a certain learning theory. Of course, it should be noted that from the beginning the development of the LO was not made according to the best methodologies, regarding its educational characteristics. This had certain didactic and technological consequences. For example, one consequence was the failure to ensure learning and acquisition of knowledge from a teacher's perspective. Also, another consequence was the inability to classify LO in terms of its educational characteristics. These characteristics were not essentially supported by the standardization bodies such as the "Institute of Electrical and Electronics Engineers" that introduced the "Learning Object Metadata" standard (IEEE LOM) for the LO's. However, this standard, although providing a description of the technical, structural and educational features of LO's along a comprehensive set of approximately 76 individual elements in order to facilitate search, evaluation, acquisition and use of LO's, it provides limited educational features. Thus, this categorization is ultimately not sufficient to represent the pedagogical and didactic concepts used in a learning environment. However, in each case, the orchestration of teaching, in other words, search and use of learning objects in order to teach a subject matter and also the selection of a learning theory is a work that is exclusively subject to teacher's choices. Technology, as we earlier mentioned in this article, does not exclude any learning theory, rather it represents an essential aid in any case, even in the case of «unwanted» behavioral methods. The crucial point lies somewhere else. It lies on the presence or absence of computer in the teaching-learning process. So, through this work we want to send a message to the teacher, who has only core skills in the use of computers. It is very important for this teacher to understand that no special knowledge to organize an ICT-supported instruction is required. And to be completely clear, hoping to convince this teacher, we will also give an example. All the educational material we need to use, it can be found ready online. Simply relying on a appropriate learning theory, we will use this material to encourage students, to engage them in the learning process, to help them develop conjectures, and to make them examine, discover, and construct their new knowledge by themselves. It should be noted that the idea of choosing Internet as a learning environment has a comparative advantage, because teacher may enrich the teaching through a wide variety of learning material that is ready-available in

different file formats such as java applets, flash animations, interactive multimedia applications and educational material ready set up with software known as "The Geometer's Sketchpad" or "Cabri". It is also important that teacher will find in the internet material beyond what is likely to be built by the same certified software, which has the necessary technological skills to create himself. So, instead of the traditional teaching tools, such as blackboard, chalk, etc. that represent a static project on which student must work, the dynamic interactive manipulation of images, shapes and values creates the appropriate conceptual structures for the student to understand the subject matter. In this way teaching becomes more effective. Therefore, the key-point, in teaching Pythagorean Theorem with the support of ICT, is that the variety of representations used, through several different educational material, provide substantial assistance to students to discover the theorem by themselves and to be convinced for its validity, through its monitoring proof.

3. The Model of p-m combinations

3.1 Van Hiele Theory
According to van Hiele Theory there are five hierarchical levels that students pass through as they progress from merely recognizing a figure to being able to write a formal geometric proof [van Hiele (1986)]. Alan Hoffer named the first level Recognition, the second level Analysis, the third level Informal Deduction, the fourth level Deduction and the fifth level Rigor [Hoffer (1981); (1986)]. Along the levels van Hiele proposed five phases to help students progress from one level to the next one. Van Hiele has called these phases Familiarization or Information, Guided Orientation, Verbalization or Explicitation, Free Orientation, and Integration.

3.2 The Cognitive apprenticeship

Cognitive apprenticeship is the application of the principles of apprenticeship to learning cognitive skills. [Collins et al. (1989)] propose in alternative model of instruction that is accessible within the framework of the typical American classroom. It is a model of instruction that goes back to apprenticeship but incorporates elements of schooling. We called this model cognitive apprenticeship. They, also, claim that Cognitive Apprenticeship *makes thinking visible*.

3.3 The Model of p-m Combinations

We attempted to teach the course of Geometry using ideas from the above theories. In detail we combined the phases proposed by the Theory of van Hiele with the methods of Cognitive Apprenticeship and we enriched these combinations with our own ideas

so that our instruction would be more coherent with Vygotsky's ideas (i.e. zone of proximal development, etc) [Dafermos, (2002)]. We have used the term "Model of Phases-Methods Combinations" or "Model of p-m combinations" to describe the proposed model.

More specifically:

Phase 1 "Information" of van Hiele's theory was combined with the method of Modeling of Cognitive Apprenticeship. This is the first combination.

Phase 2 "Bound Orientation" of van Hiele's theory was combined with the method of Coaching of Cognitive Apprenticeship. This is the second combination.

Phase 3 "Explicitation" of van Hiele's theory was combined with the method of Articulation of Cognitive Apprenticeship. This is the third combination.

Phase 4 "Free Orientation" of van Hiele's theory was combined with the method of Exploration of Cognitive Apprenticeship. This is the fourth combination.

Phase 5 "Integration" of van Hiele's theory was combined with the method of Reflection of Cognitive Apprenticeship. This is the fifth combination.

All the above phases of van Hiele's theory were combined with the method of Scaffolding of Cognitive Apprenticeship.

3.4 The role of computer in Model of p-m combinations

Computer plays an important role in the Model of p-m combinations and offers dynamic tools, which allow pupils to discover and to understand more quickly new ideas. Students, using computer, can explore shapes, discover its attitudes, develop conjectures, test theirs validity, and explore the hypotheses and possible solutions of a problem.

In *the first combination*, teacher using computers help students to accomplish their task. For example, firstly students make a Platonic solid by opaque paperboard (figure 3). Students cannot see its parts analytically, because some of them (e.g. diagonals of
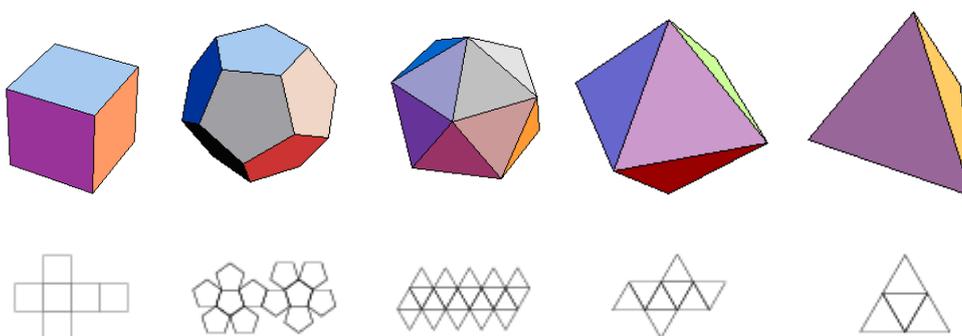

Figure 3

cube) are inside the opaque solid. In this case teacher must in advance have found the appropriate learning object from digital repositories on the Internet, i.e. a clear solid that can be rotated, folded and unfolded. Then the teacher using computer can use this see-through solid. Now, students can see the parts (diagonals) of the solid analytically, explain and name them.

In *the second combination*, computers have the ability to help students to discover attitudes of shapes. For example if students draw a cube in their papers and measure its diagonals they will not find that they are equal. This happens because they have drawn a three-dimensional object in a two-dimensional space. If they measure the diagonals of their sketch using the appropriate computer command they will discover that they are equal (figure 5).
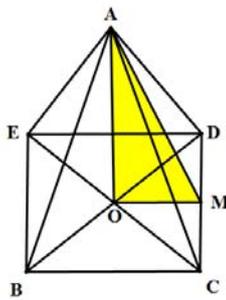

Figure 4

In *the third combination* computer give enables students to express, claim and demonstrate on screen of the computer their ideas to the rest of the classroom. For example, they can rotate a pyramid to explain that an inner triangle is a right one (figure 5)

Figure 5

In *the fourth combination*, computers challenge students to develop conjectures about the methods and strategies they use in problem solving, since they give them the opportunity to directly observe their results in the simulated situation.

In *the fifth combination*, students explain what, how, and why they did it. Students have the opportunity to analyze their performance from different perspectives and thus they may compare their views with those of experts or other classmates.

3.4 Structured Form Worksheet (SFW)

To implement this combination, we coined a special worksheet, named "Structured Form Worksheet" (SFW) [Dimakos et.al (2008)], that we briefly describe below (See Appendix).

Structured Form Worksheet (SFW)

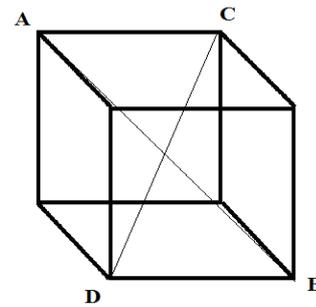The SFW consists of the following three sections :

a)   The Reminder Notes.

b) The Process.
c) The Assessment.

The Reminder Notes

In the first section, named "Reminder Notes", teacher reminds students of some theorems. These are some essential theorems, based on students' prior knowledge, which help students understand the new cognitive object. In this section, what takes place is the first combination.

The Process

In the second section, named "Process", students have to make conjectures, discover, argue, prove, and express their opinion on how to solve certain problems, that teacher has prepared for them beforehand. In this section the second, the third, and the fourth combination takes place.

The Assessment

In the third section, named "Assessment", students must tell each other what they did in the previous sections; describe the way they thought, why they thought in this way, what they learnt etc. In this section, students have to describe over the phone what they have learned to another schoolmate, who was absent from class. Moreover, students have to make a problem based on the knowledge that they have gained. In this section the fifth combination takes place.

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Appendix

Structured Form Worksheet

Instructor ……………………….

Students' full names

_____

_____

_____

Class ……………………………
School ……………………………..
Date …………………………

Lesson topic: **The Pythagorean Theorem**

1. The Reminders Notes.
Answer the following questions from the History of Mathematics using the Internet.
    1.1. Who first proved the Pythagorean Theorem?
……………………………………………………………………………………
    1.2. Which school was founded by Pythagoras and why was it dissolved?
……………………………………………………………………………………
    1.3. Which is the usefulness of Pythagorean Theorem in every day life?

(In this point students watch a video. Romeo wants to meet Juliet, who is locked in a high tower, the Capulet House. She is too high for him to climb. Therefore, he needs a ladder to reach her, but he does not know how long it should be. The length of the ladder is the hypotenuse of a right triangle that is formed by the ladder, the wall and the ground (Figure 6). Then, students see one more video clip. It demonstrates the construction of a roof with vertical sides (Figure 7)).
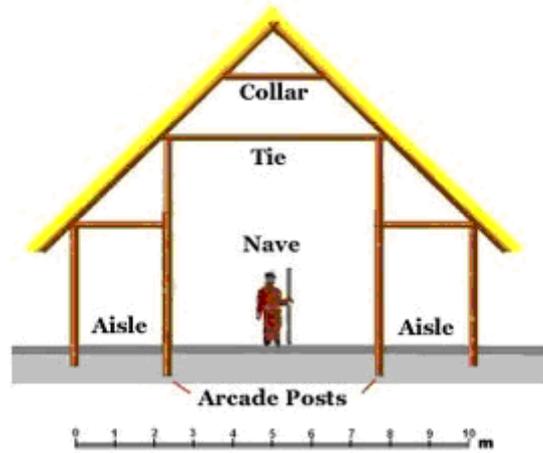

Figure 6

Figure 7

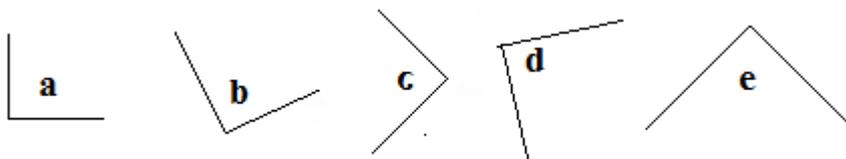1.4. A right angle is an angle of 90 degrees. You can rotate it.

Figure 8

(Students rotate the right angle on computer's screen. In this way teacher tries to help some students to overcome a cognitive obstacle. Particularly, students recognize the angle "a" as a right angle, but they have difficulty to recognize that the angles b, c, d and e are right angles, too (Figure 8). According to [Herscovics (1989a, p. 80)] errors in students' conceptualizations "are all strong indicators of the existence of cognitive obstacles").

1.5. A hypotenuse is the longest side of a right triangle, the side opposite the *right angle* (Figure 9).
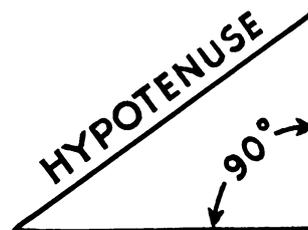
Figure 9

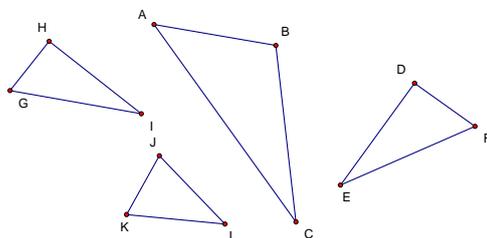1.6. The triangles GHI and EDF are right triangles (Figure 10).

Figure 10

1.7. The triangles **ABC** and **JKL** are NOT right triangles (Figure 9).
(The underlined words imply hyperlinks. In this case they open another window that explains what is not a right triangle and the kinds of triangles)

Problem Find the lenth of the part of the light path that is in the prism
(Students watch the problem in a flash animation (Figure 11).
To solve the problem first do the following activities).

2. The Process

2.1. **Activity-1** Open the file Pyth-1, that is located on your desktop, with Internet Explorer. Then describe the applet (Figure 12).
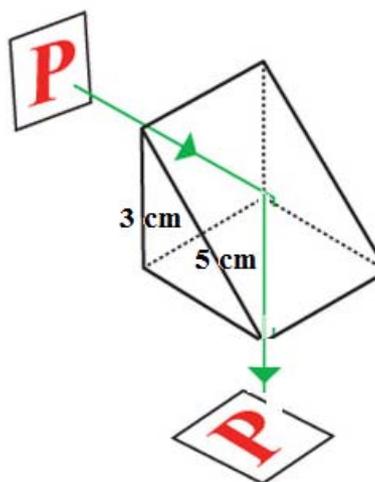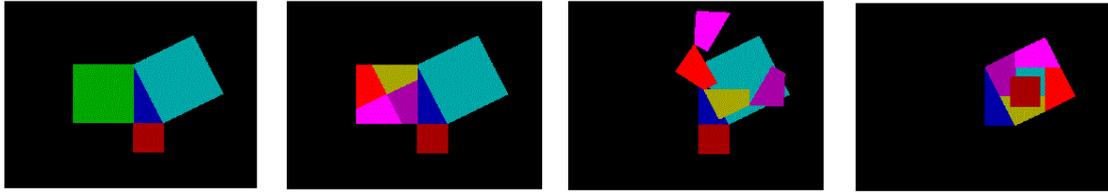

Figure 11

Figure 12

……………………………………………………………………………………

2.2. **Activity-2** Suppose that the areas of squares in this applet are 9 cm^2 and 16 cm^2. Compute the area of the square of the hypotenuse.

……………………………………………………………………………………

2.3. **Activity-3** Open the file pyth.gsp, that is located on your desktop. Move the points A, B, and C and then complete the table (Figure 13).
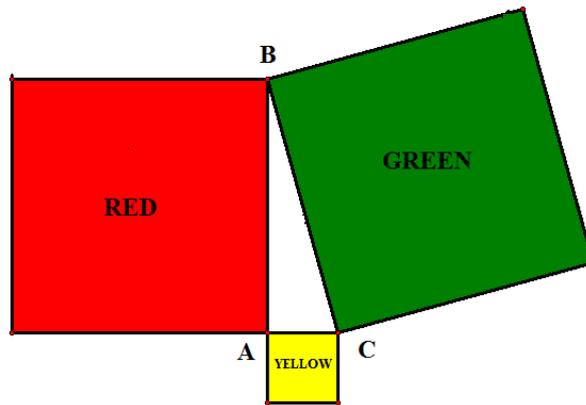

*Figure 13***:** *file of Sketchpad*

Area of red square	Area of green square	Area of yellow square

2.4. **Activity-4** Find a relationship between red, green and yellow areas

…………………………………………………………………………………………………
………………………………………………………
…………………………………………

2.5. **Activity-5** Go to page:

http://5gym-kozan.koz.sch.gr/games/pyth.htm

Watch the flash animation and discuss it with the other members of your team (Figure 15). Explain what happens in this animation. Write down your opinion.
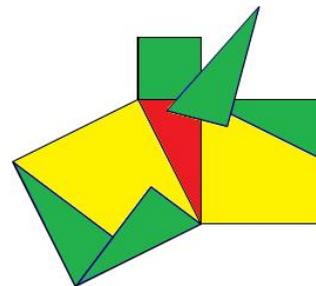…………………………………………..

Figure 14

2.6. **Activity-6** Formulate the theorem and draw the shape.
…………………………………………………………………………………………………
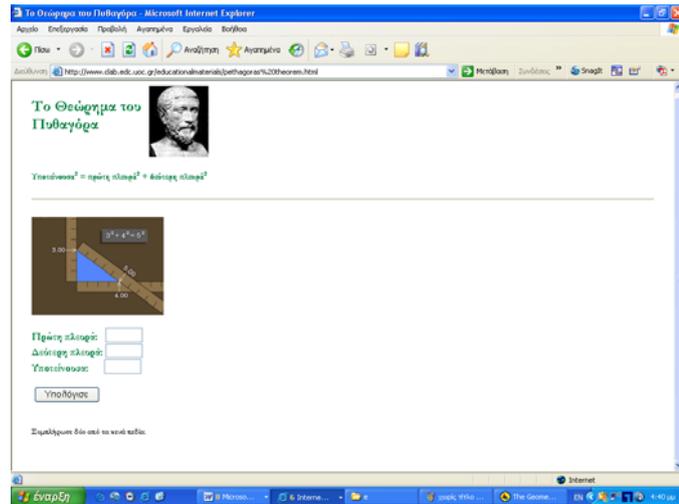…………………………………………………………………………………………………
…………………………………………………………………………………

2.7. **Activity-7** Use the Internet to find a description of the Pythagorean Theorem. Compare it with your answer in activity - 6.
…………………………………………………………………………………………………
………………………………………………………………………………………

Activity-8 Go to page:
http://www.clab.edc.uoc.gr/educationalmaterials/pethagoras%20theorem.html
Use the applet to calculate the vertical or hypotenuse of some right triangles.

(Initially, students draw a right triangle on their papers; they measure the two out of three sides of it and they calculate the third one, based on the formula they have learnt from theory. Then, students apply the Pythagorean Theorem using the applet. They fill the same two values in the blank fields of the applet and the applet calculates automatically the third unknown value. Students compare the value that they calculated with the value calculated by the applet. In this way, they can test whether they can use the Pythagorean Theorem correctly. In case the result that students found differs from the result given by the applet, students discuss this issue with their schoolmates and teacher according to Zone of Proximal Development).

3.   Assessment
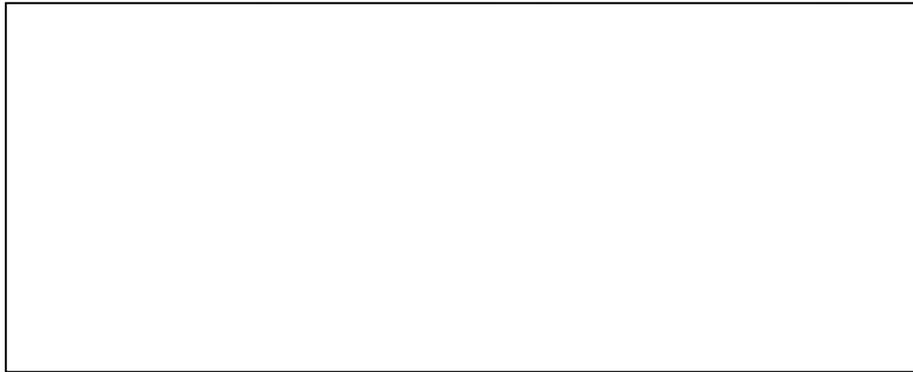 (Students answer the following questions)

3.1. What have you learnt? Draw the figure and write the theorem:

THEOREM

…………………………………………
…………………………………………
…………………………………………
…………………………………………
…………………………………………
…………………………………………
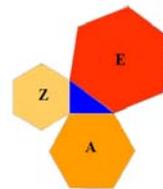…………………………………………
…………………………………………
…………………………………………
……………

3.2. Draw three triangles such that the Pythagorean Theorem is not applied.

3.3. Calculate the angles of an isosceles triangle such that the Pythagorean Theorem is applied.

3.4. Describe over the phone what you learnt to another schoolmate, who was absent from class.

3.5. Find a relationship between the areas Z,E, and A

3.6. Write a problem based on the theorem you learnt.

A joke