A Proposed Model to Teach Geometry to First-Year Senior High School Students

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Abstract
We combined the phases of the van Hiele theory with the methods of Cognitive Apprenticeship and enriched these combinations adding the three following ideas: (a) a special worksheet, named Structured Form Worksheet, which we used when teaching geometry (SFW), (b) a matrix named Reasoning Control Matrix for the Proving Process (RECOMPP), which helped students with reasoning production and (c) the concepts of simple and partial proof to write the formal proof. We called the above mentioned combination Model of p-m Combinations. Then, we used this model to teach geometry courses to 15-year old senior high school (Lyceum) students. In this article we claim that students should be able to write simple and partial proofs before they are taught how to write formal proofs.

Key-words and phrases: Formal, Simple, Partial Proofs; Phases-Methods Combinations Model; Structured Form Worksheet; Euclidean Geometry; Reasoning Control Matrix for the Proving Process.

1. Introduction
Research related to the understanding of geometric concepts by students has shown that students have difficulties in defining and recognizing geometric shapes and in the use of deductive thinking in geometry (Pyshkalo 1968; Burger, 1982; APU, 1982; Hart, 1981). Despite the importance of proofs, research has shown that students have great difficulty with the task
Due to students’ difficulty to write proofs successfully, numerous projects focus on the teaching of geometrical proof (Hanna, 2000; Martin & Harel, 1989; Leron, 1985; Recio & Godino, 2001; Senk, 1985; Usiskin, 1982). The inefficiency to teach the notion of “proof” is almost global (Hadas et al., 2000).

In Greece, Euclidean geometry is taught under a theoretical framework, during the first two years of Lyceum. Junior high-school students usually count and calculate, based on specific situations, whilst they seldom make use of abstract procedures. Thus, first-year Lyceum students, who move from specific procedures to more abstract ones, are not familiar with the role of axioms, definitions, and theorems. Instead, they have to cope with the concept of proof in a purely theoretical context. The fact that the students of Lyceum cannot learn the proof processes correctly seems to influence their future ability as undergraduates to solve mathematic problems. So, university teachers realize that the processes which first-year undergraduate students follow, when solving a mathematic problem, are the typical ones they have learnt in preparatory schools or private lessons (Kalavassis, 1996).

2. Simple proposition

According to Dimakos and Nikoloudakis (2008) a proof is constituted and is analysed in simple justifications. We develop this aspect here briefly, because this analysis represents a necessary component for this article. Initially we give two examples to explain what we mean by the words...
“statement”, “justification” and “partial proof”. Hence, let us consider the following two propositions:

Proposition (P-1): If AB=AC, prove that the triangle ABC is isosceles (See Figure 1).

Proposition (P-2): The exterior base angles of an isosceles triangle ABC are equal (See Figure 2).

Generally, we maintain that every proposition contains a statement and every proof consists of two parts, a statement (which needs a justification) and a justification (of this statement).

Especially, we can maintain that every proposition contains a statement and every proof (of this proposition) consists of two parts, a statement (the statement of the proposition which needs a justification) and a justification (of this statement) (see Figure 3).

- For proposition P-1 the statement is: the triangle ABC is isosceles.
- For proposition P-2 the statement is: the exterior base angles of an isosceles triangle ABC are equal.

It is also noted that for the proof of the statement of proposition P-1 we have:

i. (i) Statement: the triangle ABC is isosceles

ii. (ii) Justification: because AB=AC

So when we say that the triangle ABC is isosceles, because AB=AC, then we have fully reasoned the statement: the triangle ABC is isosceles for proposition P-1. Thus we have proved proposition P-1.

- For proposition P-2 the statement is: the exterior base angles of an isosce-
les triangle ABC are equal.
It is also noted that for the proof of the statement of proposition P-2 we have:

i. **statement** “the exterior base angles of an isosceles triangle ABC are equal”, and

ii. **justification** “because they are supplementary to the equal angles B and C”.

Nevertheless, we haven’t fully reasoned the statement (i) with the justification (ii) because we have not reasoned that the angles B and C are equal. As a result justification (ii) (because they are supplementary to the equal angles B and C) is a statement and is consequently a new proposition. Since any proposition in geometry, except for definitions, postulates and axioms, needs a proof, the new proposition needs a proof as well.

The statement of the new proposition is: the supplementary angles B and C are equal and its proof components are:

iii. **statement** “the supplementary angles B and C are equal”, and

iv. **justification** “because the angles B and C are equal”.

Also, we have not fully reasoned the statement (iii) with the justification (iv). Now we must explain why the angles B and C are equal. Thus, the justification (iv) is another new proposition with statement: the angles B and C are equal and its proof components are:

v. **statement** “the angles B and C are equal”.

vi. **justification** “because the triangle ABC is isosceles”.

Similarly we must explain why the triangle ABC is isosceles, so we have the proposition with the statement: the triangle ABC is isosceles and its proof components are:

vii. **statement** “the triangle ABC is isosceles”.

viii. **justification** “it is given”.

The following are observed: justification (ii) of statement (i) in proposition...
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tion P-1 does not need further justification for statement (i) to be valid so the proof is fully reasoned by (i)-(ii). However, this is not the case in proposition P-2 for justifications (ii), (iv) and (vi) of this proposition. In detail, for statement (i) to be valid, justification (ii) has to be valid. For justification (ii) to be valid, justification (iv) has to be valid and so on. So with just (i) and (ii) the proof of P-2 is not complete.

When the justification of a statement, like in proposition P-1, does not need further justification for the statement to be valid, then the justification is referred to as simple justification. In particular:

**Definition:** A justification is called *simple justification* when no further justification is needed in order to prove its truth. Simple justification will also be called *simple proof*.

**Definition:** We shall say that a justification of a statement is a non-simple justification or a partial proof when its truth depends on yet another justification.

Proofs (i)-(ii), (iii)-(iv), (v)-(vi) of proposition P-2 are non-simple justifications but the last part of the proof of P-2, that is proof (vii)-(viii), is indeed a simple justification. To conclude, the proof of P-2 consists of some partial proofs (i)-(ii), (iii)-(iv),(v)-(vi) and a simple justification (vii)-(viii).

Taking the above in to account we can say that every proof is consists of two components. These parts are both a statement and a justification, or a statement and a justification, which corresponds to a partial proof (see Figure 4). Nevertheless, the partial proof is a proof itself. So, it can be further analysed to a simple justification or to a partial proof and

![Figure 4](image-url)
so on. This way a proof consists of and is analysed in simple justifications (see Figure 5).

Also, we define as simple proposition a proposition whose proof is a simple proof. In this article it is claimed that students should be able to write simple and partial proofs before they are taught how to write formal proofs.

3. Two well known theories

Dutch educators Pierre van Hiele and Dina van Hiele-Geldof developed a theory of five levels of geometric thought (Anderson, Reder and Simon, 1996). According to the van Hiele theory there are five hierarchical levels that students pass through as they progress from merely recognizing a figure to being able to write a formal geometric proof. Alan Hoffer (1981), named the first level Recognition, the second level Analysis, the third level Informal Deduction, the fourth level Deduction and the fifth level Rigor. Along the levels van Hiele proposed five phases to help the students progress from one level to the next one. Van Hiele has called these phases Familiarization or Information, Guided Orientation, Verbalization or Explicitation Free Orientation, and Integration.

Cognitive apprenticeship is the application of the principles of apprenticeship to learning cognitive skills. Collins et al. (1989) comment:
We propose in alternative model of instruction that is accessible within the framework of the typical American classroom. It is a model of instruction that goes back to apprenticeship but incorporates elements of schooling. We call this model cognitive apprenticeship.


They, also, claim that Cognitive Apprenticeship makes thinking visible. We believe that this aspect helps a mathematician teach the students to write successful proofs in geometry.

In our research we attempted to teach the course of Geometry using ideas from the above theories. In detail we combined the phases proposed by the Theory of van Hiele with the methods of Cognitive Apprenticeship and we enriched these combinations with our own ideas so that our instruction would be more coherent with Vygotsky’s ideas (i.e. zone of proximal development, etc). We have used the term “Model of Phases-Methods Combinations” or “Model of p-m combinations” to describe the proposed model.

3.1. Model of Phases-Methods Combinations

Regarding the analysis of phases, van Hiele (1986, p. 177) mentions the following: “I have not mentioned a specific form of instruction. The ideas that have been used here have a place in every method of teaching”. Also, Collins et al. (1991) believed that there are more than one ways to apply the methods of Cognitive Apprenticeship and that, ultimately, the teacher is the one who is responsible for determining the ways in which cognitive apprenticeship can be applied in the range of his/her teaching.

We attempt to teach the course of Geometry combining the phases proposed by the theory of van Hiele with the methods of Cognitive Apprenticeship taking into consideration the above statements of van Hiele (1986 p. 177), of Collins et al. (1991) and also that:
Students find it difficult to understand the course of geometry and its processes (Van Hiele, 1986; Hoffer, 1981; Usiskin 1982; 1987; Burger and Shaughnessy, 1986; Crowley 1987; Fuys, Geddes, and Tischler 1988; Gutierrez, Jaime, and Fortuny 1991; Mason 1997; Wirszup, 1976)

Students find it very difficult to successfully write simple geometry proofs (Weber, 2003). This happens, when they repeat proofs taken from their coursebook as well (Burger & Shaughnessy 1986; Hoffer 1983; Wirszup, 1976)

The van Hiele theory of levels of geometric thought specially refers to the course of Geometry

Cognitive Apprenticeship, according to its creators Collins et al. (1989) and Collins et al. (1991) makes the thought visible

According to Fuys et al. (1988) the progress from one level to another depends on the teaching method followed by the instructor, regardless of the age of the students or their biological maturity

The combination of the phases of instruction of the van Hiele theory with the methods of Cognitive Apprenticeship was based on the participants’ characteristics, actions, and roles in the teaching process in both theories.

More specifically:

**Phase 1** “Information” of van Hiele’s theory was combined with the method of Modeling of Cognitive Apprenticeship.

**Phase 2** “Bound Orientation” of van Hiele’s theory was combined with the method of Coaching of Cognitive Apprenticeship.

**Phase 3** “Explicitation” of van Hiele’s theory was combined with the method of Articulation of Cognitive Apprenticeship.

**Phase 4** “Free Orientation” of van Hiele’s theory was combined with the method of Exploration of Cognitive Apprenticeship.

**Phase 5** “Integration” of van Hiele’s theory was combined with the method of Reflection of Cognitive Apprenticeship.

All the above phases of van Hiele’s theory were combined with the method of Scaffolding of Cognitive Apprenticeship.
3.1.1. SFW and RECOMPP

To implement this combination we coined a special worksheet, named “Structured Form Worksheet” (SFW). (Dimakos, Nikoloudakis, 2008). The SFW and an important component of SFW (Dimakos, Nikoloudakis, 2007) called “Reasoning Control Matrix for the Proving Process” (RECOMPP) are briefly described below.

3.1.1.1. Structured Form Worksheet (SFW)

The SFW consists of the following three sections:

a) The Reminder Notes.

b) The Process.

c) The Assessment.

The Reminder Notes

In the first section, named “Reminder Notes”, the teacher reminds the students of some theorems. These are some essential theorems, based on the students’ prior knowledge, which help students understand the new cognitive object. In this section, what takes place is the combination of the first phase of van Hiele’s model (Inquiry/Information) with the method of Modeling of the Cognitive Apprenticeship model.

The Process

In the second section, named “Process”, students have to conjecture, to discover, to argue, to prove, and to express their opinion on how to solve certain problems, that the teacher has prepared for them beforehand. In this section the following combinations take place:

• the combination of the 2nd phase of van Hiele’s model (Directed Orientation) with the method of Coaching of the Cognitive Apprenticeship model

• the combination of the 3rd phase of van Hiele’s model (Explication) with
the method of Articulation of the Cognitive Apprenticeship model
• the combination of the 4th phase of van Hiele’s model (Free Orientation)
with the method of Exploration of the Cognitive Apprenticeship model.
In this combination students make use of a matrix that we coined,
dubbed “reasoning control matrix for the proving process” (RECOMPP).

The Assessment
In the third section, named “Assessment”, students have to tell each
other what they have done in the prior section, they have to describe the way
they have thought, why they have thought this way, what they have learned
etc. In this section, students have to describe over the phone what they have
learned to another schoolmate, who was absent from class. Moreover, stu-
dents have to construct a problem based on the knowledge that they have
gained. This section constitutes of the fifth phase of van Hiele’s model (In-
tegration) with the method of Reflection of the Cognitive Apprenticeship
model.

3.1.1.2. RECOMPP
According to Dimakos and Nikoloudakis (2007) RECOMPP is a reus-
able matrix pattern that helps students produce reasoning production. Its
layout and its filling technique are predefined. In more detail, it consists of
six discrete sections and its layout consists of rows, columns, and cells that
may contain figures, hypotheses or conclusions, proofs, and partial proofs
(see Figure 6). Furthermore, when filling RECOMPP, a student follows two
basic rules: that of horizontal transit, and that of transfer. These rules will be
described in more detail later in this article. RECOMPP can be used in
every chapter of geometry content because it is a reusable pattern of reason-
ing production. The advantage RECOMPP offers, when employed by nov-

dice lyceum geometry students attempting to prove a proposition, is that it
can help them produce and control their reasoning in a more effective way
so as to successfully write the given proof (Dimakos, et. al, 2007).

As shown in figure 6, RECOMPP consists of six discrete sections. What follows is a detailed description of each section:

- **Section 1**, is where the formulation of the problem is given. Here, in a textbox, the student can read the full description of the problem, before he/she moves on to the proving procedure.

- **Section 2**, is where the hypotheses, and the conclusions of the problem must be written. Here, the student is given a table (consisting of two rows and two columns), where he/she must write down, in two separate lines, the hypotheses, and the conclusions of the problem, respectively. Of course, students must have very carefully read the description of the problem, that can be found in Section 1, very carefully, before they are able to find, discriminate, and record the hypotheses.

- **Section 3**, is where the sketch of the problem must be prepared by the student. Here, based on the description of the problem they have read in Section 1, and according to the hypotheses, and the conclusions that they have written down in Section 2, student progress to draw the sketch of the problem in a blank field. Students will use this, as a visual aid, to write the proof.

- **Section 4**, is where the teacher may offer scaffolding to student. Here, in order to offer students contextual, and on the spot help, the teacher can provide them with a list of hints.

- **Section 5**, is where the student is motivated to reason, collect, and write those statements and relationships between the elements of the sketch, prepared beforehand, which will lead him/her to the successful writing of the proof. Here, the student is given a table (consisting of just two columns and several rows). In this table, in the first column, the student must write a statement e.g “Statement A”, that needs to be proved, labeled “To prove that…”. In the second column, the student must write a statement e.g “Statement B”, that is necessary in order to prove “Statement A”, labeled “It
is required to prove that”. The student must set some goals, in order to move horizontally and proceed from the left column to the right one. Thus, the left column represents the earliest stage of the student’s reasoning and
the right column represents the latest stage of the student’s reasoning. Section 5 of RECOMPP must be filled according to the following three rules: (See Figure 7)

(a) the rule of horizontal movement from left to right, i.e the student first fills the left column, labeled “to prove that”, and then continues to fill the right one, labeled “it is required to prove that”. The most significant contribution of this rule of horizontal movement from left to right is that it is demanded from the student to produce reasoning. This process is repeated in every row of the RECOMPP.

(b) the rule of how to fill the first cell of the left column, labeled “to prove that”. According to this rule, the student must always fill the first cell of the left column with the conclusion from Section 2. This is especially important for the student, because, it shows him/her, where to start the proving procedure from.

(c) the rule of reassignment of produced reasoning. According to this rule, the content of the right column in each row (produced reasoning), is reassigned to the left column of the row below.

-Section 6, is where the proof must be written by the student. Here the student must write
the proof in a textfield.

3.1.2. The periods of instruction in model of m-p Combinations

According to this model the instruction takes place in five periods (see Figure 8).

In the 1st period, students relate the visual geometric shapes and their appearance with their names for every cognitive subject, e.g. of all kinds of parallelograms and their appearance with their names. Moreover, the teacher demonstrates shapes that are gradually increasing in complexity. The students are acquainted with more complex shapes and their components.

In the 2nd period, students are taught the attributes and the relative theorems of the cognitive subject (parallelograms), without their proofs. Students confirm the validity of theorems in an experimental way, using computers. It must be noted that dynamic representations that result from dynamic geometry software (DGS) environments (particularly Geometer’s Sketchpad and Cabri II+) play a semantic role as they aim to develop spatial sense and geometric reasoning (Kalavassis, Meimaris, 1996; Mariotti, 2003).

In the 3rd period, students classify the shapes (all kinds of parallelograms) and expand the properties of the shapes. For example, the properties of parallelogram are inherited to rectangle and rhombus and from them to the square.

In the 4th period, students deal with simple geometric propositions and use RE.CO.M.P.P to write the proof.

In the 5th period, students learn the proofs of all theorems (of parallelograms).

In every period (see Figure 9) students use the special worksheet mentioned before, named “Structured Form Worksheet” (SFW) which the teacher has prepared beforehand to teach a cognitive object (see appendix).
Figure - 8
FOR EVERY PERIOD $N$, $N=1,2,3,4,5$

**Figure - 9**

- **Combination-1**
  - The student familiarizes him/herself with the object

- **Combination-2**
  - Discover conjecture
  - Realises empirically properties of shapes

- **Combination-3**
  - Converse
  - Formulate

- **Combination-4**
  - The student is involved in more complex work

- **Combination-5**
  - Reflect

**Scaffolding**

- **Reminders**
  - Notes

- **Process**

- **Assessment**

**Cognitive Object**

**SFW**
4. Method

4.1. Participants and Procedure

The participants in the research were students, who were studying at the first class of High School. The method of random sampling was used to constitute the sample of the research. Overall, 250 students participated in the research. They came from five public and one private school. The majority of students originated from families of low socio-economical status. These students had never been taught the course of geometry in a theoretical context before. Therefore they were considered to be novice in theoretical geometry. Most of the students had some experience in the use of computers. Students participated in the research as whole classes, based on the distribution of classes already made by the school principals. 138 students formed the experimental group and 112 students formed the control group.

4.2. The instruction

Unit 5 from the textbook, referring to parallelograms, was taught. The instruction took place in five periods that lasted two-months period. In the 1st period, students related the visual geometric shape of all kinds of parallelograms and its appearance to their names. In the 2nd period, students were taught the attributes and the relative theorems of parallelograms, without their proofs. Students confirmed the validity of theorems in an experimental way, using computers. In the 3rd period, students classified the shapes of all kinds of parallelograms. In the 4th period, students argued simple geometric propositions and used RE.CO.M.P.P to write a proof. In the 5th period, students learned the proofs of all theorems of parallelograms.

4.3. Instruments

4.3.1. SFW

The students were taught the unit 5 from the textbook that refers to parallelograms. As we mentioned above the students used the Reasoning Control Matrix for the Proving Process. The RE.CO.M.P.P was employed for
the proof of propositions that were assigned to the students, since a theoretical document was required.

4.3.2. Proof-writing evaluation exercises

Two pairs of exercises, that were similar to those of the textbook used in class, were given successively during the pre-test and the post-test. (Totally, 4 exercises per test). The first exercise from each pair corresponded to a simple proof and the second one corresponded to a complex proof, which was an extension or a slight modification of the first exercise. We asked students to prove another one result, whose proof was based on the result of the first exercise.

The following exercises comprised the pre-test:

**Exercise 1:** In an isosceles triangle ABC, with AB = AC, the points M, N lie on the line segments AB, and AC respectively, so that M is the mid-point of AB, and N is the mid-point of AC. We equally extend the base BC of the triangle by the line segments BD, CE, so that BD=CE. Prove that DM=EN (See Figure 10).

**Exercise 2:** In an isosceles triangle ABC, with AB = AC, the points M, N lie on the line segments AB, and AC respectively, so that M is the mid-point of AB, and N is the mid-point of AC. We extend the base BC of the triangle through B and C, respectively, to points D and E, so that BD = CE. Segments DM, and EN intersect at point I. Prove that segment AI is a bisector of angle A (See Figure 11).

**Exercise 3:** Prove that the common external tangents BC and B’C’, of two externally tangential circles K and O are equal (See Figure 12).
Exercise 4: Given that BC and B’C’ are common external tangents of two externally tangential circles K and O, prove that B’C = BC’ (See Figure 13).

The following exercises comprised the post-test:

Exercise 1: In parallelogram ABCD, the points M, N lie on the line segments AB and CD, respectively, so that M is the midpoint of AB, and N is the midpoint of CD. Prove that ANCM is a parallelogram (See Figure 14).

Exercise 2: In parallelogram ABCD, the points M, N lie on the line segments AB and CD, respectively, so that M is the midpoint of AB, and N is the midpoint of CD. Prove that the diagonal AC is trisected by the segments MD and NB (See Figure 15).

Exercise 3: In square ABCD, the point E lies on the segment AB. From point A we draw a perpendicular to DE, which intersects BC at point Z. Prove that DE = AZ (See Figure 16).

Exercise 4: In square ABCD, two vertically intersected line segments intersect sides AB, BC, CD, and DA of the given square at points E, Z, E’ and Z’, respectively. Prove that EE’ = ZZ’ (See Figure 17).

Each exercise in the pre-test and the post-test was graded in a scale between 0-and 5 grades as follows:

<table>
<thead>
<tr>
<th>Grade</th>
<th>Analytical explanation of student’s actions</th>
</tr>
</thead>
<tbody>
<tr>
<td>0 -</td>
<td>The student had not written anything or had not made any valid simple propositions</td>
</tr>
<tr>
<td>1 -</td>
<td>The student had written the given input and the objects of the exercise and had also drawn the shape in the corresponding sections of RECOMPP</td>
</tr>
<tr>
<td>2 -</td>
<td>The student had written at least a simple proposition along with</td>
</tr>
</tbody>
</table>
4.4 Results

Initially, an independent-samples t-test among the experimental and the control group was conducted. The statistical test was conducted to check the existence of statistically significant differences in the performance between the pre-test and the post-test. No statistically significant difference was found in the performance in proof-writing among control group and experimental group, between pretest and post-test ($t = -1.896$ df = 248, $p > 0.05$).

Also, a paired-samples t-test among students in the control group was conducted between pre-test and post-test. This statistical test was conducted to check the existence of statistically significant improvement in the proof-writing performance of these students. No statistically significant difference was found in the proof-writing performance among students of control group between pre-test and post-test. ($t = 0.711$ df = 111, $p > 0.05$).

Finally, a paired-samples t-test among students in the experimental group was conducted, between pre-test and post-test. This statistical test was conducted to check the existence of statistically significant improvement in the proof-writing performance of these students. It was found that there is a statistically significant difference in the proof-writing performance among students of the control group between pre-test and post-test ($t = -48.271$ df = 137, $p < 0.05$).

Discussion

The findings suggest that students of the experimental group, who had employed the Structured Form Worksheet that contains RE.CO.M.P.P and had initially dealt with simple geometric propositions had significantly im-

3 - The student had written some thoughts in the “reasoning development” section of RECOMPP
4 - The student had written at least a partial proof
5 - The student had written a complete proof
proved their ability in writing formal geometry proofs, compared to the students of the control group who had been taught in a traditional method. The exercises given to the students were common exercises, that is, exercises taken from the textbook. These exercises were given in pairs. The second exercise represented an extension of the first one. Thus, the proof of the second exercise was an extension of the proof of the first exercise. We intentionally left the students uninformed of this property between the two exercises. We did so, because we wanted to examine either, whether a student, who had already solved the first exercise had the ability to solve the second exercise too, or whether a student had the ability to solve only the first exercise and not the second one. We found that most of the students in the control group could not solve the second exercise, while most of the students in the experimental group managed to solve both exercises. The above findings allow us to claim that students should be able to write simple and partial proofs before they are taught how to write formal proofs.

References
Burger, W., Shaugnessy, M., (1986). Characterizing the van Hiele levels of development in geometry, Research in Mathematics Education, Vol. 17. No 1, 31-48

HMS i JME, Volume 2, 2009 (17-45)


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Studies in Mathematics, 44, 5-23.


Pyshkalo, A., (1968). Geometry in Grades 1-4: Problems in the formation of geometric conceptions in pupils in the primary grades, Moscow: Prosres-


Appendix

Structured Form Worksheet

Students’ full names

Instructor

Class ...........................................

School  .........................................

Date ............................................

Lesson topic:  **The diagonals of a rectangle**

1. **The Reminders Notes.**

1.1. The perpendicular bisector (k) of a line segment AB is a line that is perpendicular to AB and passes through the midpoint M of segment AB.

1.2. The perpendicular bisector of a line segment is the locus of all points that are equidistant from its endpoints, i.e. GA = GB for every point F of the perpendicular bisector.
1.3. If a straight line (h) is perpendicular to one of two parallel lines (d), then it is also perpendicular to the other line (e).

\[
\begin{array}{c|c|c}
(h) & (d) & (e) \\
\hline
A & \_ & B \\
\end{array}
\]

If \( A = 90^\circ \) then \( B = 90^\circ \)

1.4. **Problem:** Two cities A and B are equidistant from points D and C of the national road n respectively. Where should a station S be built so that the points A, B, C and D are equidistant from S?

2. **Process**

(The students work in the computer. The students with the help of command measurement fill a table)

2.1. What shape is the ABCD

2.2. Measure the length of every segment SA, SB, SC, SE. Drug the point S, repeat the measurement and fill the following table.
2.3. Draw “all points” that are equidistant from A and B, and then draw all points that are equidistant from A and D (The students draw the following shape)
2.4. Can you guess where point S lies?

2.5. Write down your observation (The students discover where the point S lies)

2.6. Prove that points A, S, C are on the same line and then prove that B, S, D are on the same line too. (The students employed the RE.CO.M.P.P)

2.7. Prove that AC = BD. (The students employed the RE.CO.M.P.P)

2.8. Formulate the relation. Then write it in words:

(The students discuss the theorem and then write it down)

3. Assessment

(The students replay the following questions)
3.1. What have you learnt? Draw the figure and write the:

THEOREM

................................................
................................................
................................................
................................................
................................................
................................................
................................................

3.2. Describe over the phone to another schoolmate, who was absent from class, what you learned.

3.3. Write a problem based on the theorem you learnt

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