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THE NUMBER ANALYSIS APPROACH IN 6-YEAR-OLD CHILDREN

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Abstract

The aim of the present study is to propose a new approach of the number concept, which emphasises on the analysis of number and evaluates the children's performances regarding the solution of narrative problems of adding type. The findings of an experimental research, which was carried out on 6-year-old children is presented and discusses. These particular findings verify the hypothesis that the infants who attended the "approach of number analysis" (research team) have demonstrated a more mature ability of solving narrative problems of addition and subtraction than those who followed the usual teaching of the logical-mathematical approach (control group). Moreover, the former group has been able to apply the knowledge of analysis, especially through the analysis-synthesis use of finger patterns in many aspects of the number concept. Therefore, they demonstrated a more mature way of thinking than the children in the control team. All in all, the above results can withstand time as it was proved by the experiment using the same children as subject when they reached the 1st class of elementary school. According to the findings of this research we can conclude that the rich network of ideas that is connected with the numbers maintains and simultaneously exceeds the achievements of counting. While the children are absorbed in activities of counting, they don't think that the counted collections may be composed by two composite parts. The flexible analysis and recombining of numbers help the use of reasoning strategies and work against memorization.

***Keywords**:* number analysis, counting, addition and subtraction problems.

MSC Classification: 97D40.

1 INTRODUCTION AND THEORETICAL BACKGROUND

Most didactical approaches of numerical notions in pre-school don't give importance enough on the analysis of number. The previous researches lay emphasis on the memorization of the operations (Thorndike, 1922) or the logical and mathematical structures (Piaget & Szeminska, 1952). The latest instructive approaches of early number concepts, which were formed mainly in the USA during the 80's and 90's, and despite their differences, they have the oral numeration and the counting strategies as a starting point (Gelman & Gallistel, 1978 – Clements, 1984 – Fuson, 1988 – Hughes, 1986 – Wright et *al.* 2007).

However, the number concept is connected with a variety of analysis in smaller numbers and its reconstruction from them. The analysis and synthesis of numbers are theoretically supported in the Piaget and Resnick's works. The interpretation of numbers with terms of parts and wholes was characterized by Resnick as the biggest mental achievement during early school years (Resnick, 1983). Our perception supports an interpretation of the number, which derives partly from Piaget's definition of the operational significance of number concept (Piaget & Szeminska 1952). Piaget's definition of the number presupposes that the children are able to approach the analyses of given number even if this definition does not presuppose that the children know explicitly these analyses. In addition, Payne and Rathmell (1975) proposed the use of the "whole" and "parts" terms in order to give emphasis on this significant relation of separation. Various researchers have shown direct or indirect interest in approaches that are related to the decomposition and composition of numbers (Sophian & Corgray 1994 – Irwin, 1996 – Hunting, 2003 – Cobb et *al.*, 1997).

The main reasons for establishing the importance of number analysis on the structure and development of children's numerical knowledge are the following: the multiform and open approach, the invention of reasoning and calculation strategies, the preparation of place value and the solution of problems (Kosyvas, 2010b – Baroody, 2004). We consider that an appropriate group of instructive activities could cause and prepare some of the previously mentioned abilities. In our research program, the analysis and composition of numbers constitute a fundamental trait for the mathematic development of children. It penetrates into the comprehension of the number concept as well as of the addition and subtraction. For the comprehension of these operations, the fact that numbers can be separated in smaller ones or be combined in bigger ones is of utmost importance. In each numerical composition, the numbers that are analyzed and composed, codetermine each other, so that a number can result from others through addition or subtraction. This knowledge is connected with the "part-whole" schema and plays an important role in the solution of addition and subtraction problems (Riley & Greeno, 1988), since it is primarily inherent in the depth of conceptual relationships of each problem.

We studied narrative problems of addition and subtraction. The pupils that took part in the research solved 10 story problems in total. The question which occupies us in the present work is: *Will an instructive approach, on the basis of number analysis, bring about a deeper and more mature comprehension towards the solution of narrative problems by six-year-old infants in comparison with the usual teaching? Moreover, which strategies are preferred in each case*?

2 METHOD

The experimental planning includes two groups of children, who attend the second class of kindergarten: the experimental group (or research group) and the control group. The research group attended a new approach (syllabus) of the number concept, which puts emphasis on the analysis of number. On the other hand, the control group attended the usual method of the number approach, with activities of logic type - such as classifications, corresponding or serial ordering - as a key basis.

Activities that were thought (research group) are the following: activities of partitioning, matching and regrouping by using fingers and other suitable teaching aids that culture provides us with (small collections, parts of the counting board, structured ten, dice, plates of analysis, educational games) and familiar situations of reflection on the teaching aid or other expressive means (verbal recitation of the number word sequence, counting, narrative stories, songs and other multisensory means of the numbers approach), (Kosyvas, 2001).

Ten narrative problems, divided into five types, were given to the total sample children (i.e. two problems for each type). Our examination is limited to 6 problems of change and 4 problems of combination (Riley & Greeno, 1988).

The numbers that are included in these narrative problems, both as given numbers and as asked results do not exceed number 8. The table below presents the list of narrative problems given to the children (other numbers that are mentioned in the same scenario are given in brackets):

Table 1. – Description of the list of narrative problems

Categories	Dictation of the 10 Problems
Change 1: 2 +6=x (3+4=x), increase with situation as an unknown quantity	George had 2 (3) markers. For his drawing he want more colours. So he is looking for more markers and h (4). How many markers has George got now?
Change 2: 8–6=x (8–4=x), decrease with situation as an unknown quantity	Grandfather had 8 (8) hens in the henhouse. However, a fox got in and ate 6 (4) hens. How many hens remain henhouse?
Change 3: 3 +x=7, (2+x=8) increase with t quantity as the unknown	There were children 3 (2) on the school bus and a few on it too. Now there are 7 (8) children on the bus. Ho children got on the school bus?
Combine 1: 2+4=x (5+2=x), finding of the	Nick has got 2 (5) fluffy toys (kittens) and 4 (2) fluffy How many animals has Nick got?
Combine 2: 2+4=x (5+2=x), finding of the	At Melina's birthday party there were 7 (8) children. them were girls. How many were the boys?

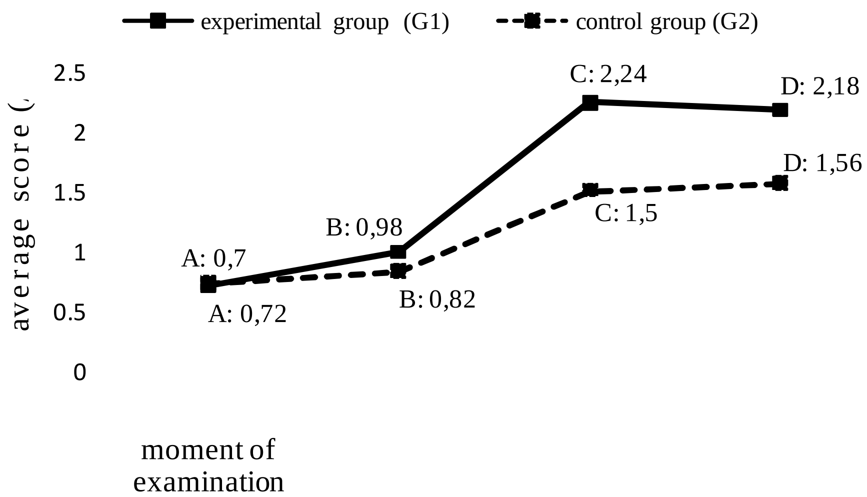
All children were asked the questions of the ten narrative problems at four different phases: before the application (pre-test or A), after the first phase of courses (test B), after the expiry of all courses at the end of school year (test C) and at the beginning of the new school year in the first class of primary school (post-test or D).

The experimental research was carried out during the school year 1992-93 in 22 kindergarten classes with a total of 258 pupils. More specifically: During the measurement (in October), our instructive method was used for teaching a hundred and twenty nine (129) children (75 boys, 54 girls) from 11 classes of the Corinthian prefecture (Greece), aged from 5*6 (5 years and 6 months) to 6*6 (average=6.02, while 51,9% of the children were older than 6). Additionally, during the last measurement a hundred and twenty nine (129) children (61 boys, 68 girls) from other 11 kindergarten classes, aged from 5*6 of (5 years and 6 months) to 6*6 (average=6.06, while 63,6% of children were older than 6) were taught the approach of number concept as this is described in the official curriculum of kindergarten. The measurement scale of the children's performances on all problems ranged from 0 to 4. The statistical package (SPSS) was used for the hypothesis control and the inductive statistical data analysis.

3 PRESENTATION AND DISCUSSION OF RESULTS

The following diagram depicts the comparative development of average performances of both groups in the 10 narrative problems at phases A, B, C and D. Moreover, the parallel line control of the diachronic development of both groups is carried out.

Fig. 1. – Performances of both groups in the 10 narrative problems in relation with the moment of the examination.



Both during the implementation of the programme (A-B-C) and after its completion (D), the average performances of the experimental group are higher than the performances of the control group, as far as these particular problems are concerned. In other words, the experimental group exhibits more progress. Comparing the diachronic development of the performances of both groups in the narrative problems (hypothesis control of both parallel lines), it is proved that the interaction of diachronic evolution within the subjects with the group type is statistically important (GLM Repeated Measures-Sphericity Assumed: $F=30.81$, $p<0.01$).

In addition, these results are long lasting, as this was proved with the repetition of the test when the children were in the first class of elementary school. This superiority of the experimental group (G1) against the control group (G2) should be attributed to the unique specific difference between both groups: particularly, the special instructive intervention that builds on the children's previous numerical experiences, the use of familiar and experientially attractive teaching aids and also the partition-composition of numbers.

The results of the research brought to light the abilities of preschool children in the solution of problems, their difficulties and also the rich variety of their strategies. Below we will attempt to analyze these subjects. The table billow provides information on the rates of success of both groups in each of the 10 narrative problems and it allows us to formulate the following observations:

Table 2: Diachronic evolution of the total success rates
of both groups in the narrative problems

Description of the narrative problems	G	A	B	C	D
1. Increase with unknown the final	(	8.	14.	72	70.
(2+6=x)	(	9.	11.	38	41.
2. Increase with unknown the final	(	12	24.	78	79.
(3+4=x)	(	13	15.	40	41.
3. Decrease with unknown the final	(	11	23.	77	72.
(8–6=x)	(	10	15.	35	40.
4. Decrease with unknown the final	(	10	24.	79	75.
(8–4=x)	(	12	16.	46	48.
5. Increase with unknown the added	(	7.	17.	53	52.
(3+x=7)	(	10	12.	32	34.
6. Increase with unknown the added	(	6.	14.	49	49.
(2+x=8)	(	7.	9.	31	33.
7. Finding of the whole (2+4=x)	(	11	23.	82	76.
	(	12	15.	45	48.
8. Finding of the whole (5+2=x)	(	10	19.	78	73.
	(	10	13.	43	46.
9. Finding of the part (7=4+x)	(	10	23.	67	62.
	(	10	16.	35	34.
10. Finding of the part (8=6+x)	(	9.	21.	63	61.
	(	9.	15.	34	35.

A first finding is that in each of the measurements (B, C, D) the success rates among the children of the experimental group are higher than the respective rates of the control group for all problems. The children were proved to be capable of solving a wide spectrum of problems with narrative stories. Their performances were high. In particular, when the children started attending the first class of elementary school (in October), the percentage of success ranged from 49.6 to 79.1 in the experimental group and from 33.3 to 48.8 in the control group.

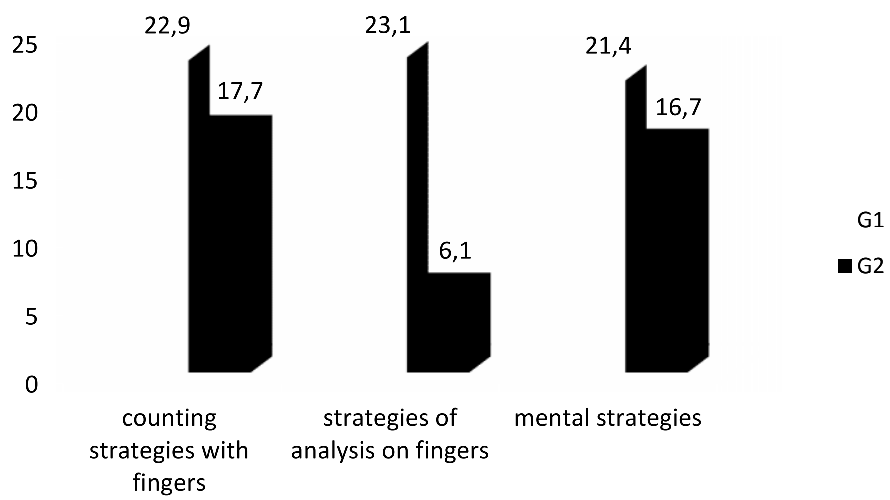
A second finding concerns the effect of the semantic structure in the difficulty of problems: problems that differ in the semantic structure, but are solved with the same numerical operation, differ in the degree of difficulty. This conclusion agrees with the findings of other researches (Riley et *al.*, 1983). Despite the fact that each problem constitutes a separate case, these problems can be divided into two categories: the easy problems (of increase or decrease with the final situation as the unknown quantity and the finding of the whole), in which the children of the experimental group presented rates of success from 70.5 to 82.9 and the difficult problems (the second quantity, finding of part), in which the rates of success for the experimental group were 49.6 to 67.4.

The children comprehended easy problems of change (increase or decrease) with unknown the final situation and problems of combination with the whole as an unknown quantity. In a like manner, they face problems of combination, which are reported in certain familiar classes that are enclosed in a total class.

The children had difficulties with the problems of the second category (missing added problems, finding of part). The major difficulties mainly related to the comprehension of these problems. In addition, the children found it difficult to represent the data with their fingers. The relationship between 'part'-'whole', which is inherent in the semantic structure of these problems, is of different nature. However, the children's performances are high, mostly due to the application of a program of Mathematic activities that placed emphasis on the numbers' relationships of decomposition-composition. Finally, they made progress passing from solutions of problems with direct material representations (by using their fingers) to the use of strategies concerning the analysis and recombining of numbers (Sophian & McCorgray 1994).

However, from the presentation of the findings, we can conclude that in narrative problems, the children of the experimental group excel not only at the total rate of success, but also at the type of strategies.

Fig. 2.– Rates of correct strategies in the 10 problems after the program (D)



The strategies selected by the children of the experimental group are more mature than the corresponding ones of the control group. This can be seen in the increased percentages in all strategies and also in the big superiority of the experimental group in the strategies of analysis on fingers (G1: 23.10, G2: 6.13), that recommend the intermediary pre-mental stage for the passage from processes with objects to strategies without objects. The supremacy of the experimental group (G1) is explicit (G1: 21.40, G2: 16.68) in every test concerning mental strategies. The same thing happened with the counting strategies with fingers (G1: 22.88, G2: 17.68) (Kosyvas, 2001). We can conclude that the familiarization of the children with the analysis and recombining of the finger patterns help in the progressive formation of the number concepts. However, each child does not show consistency concerning the use of strategies. The strategies differ both between problems and also within the same problem. The children frequently change their strategy during their effort to give an answer to the problem. They mobilize various strategies from the ones they know or they re-establish their familiar strategies and construct new.

4 CONCLUSIONS

Summarizing the above findings, we conclude that the children of the experimental group, who had been taught the number concept according to the approach of analysis, demonstrated a wide and composite capacity for problem solving. Our findings show that the children of the experimental group exceed the level of abilities determined by the curriculum aimed for the first term of the first class of primary school.

The instructive approach in the kindergarten according to the official Curriculum (control group) presents the following characteristics: persistence in the pre-requisite logic concepts (classifications serial-ordering), depreciation of the importance of numbers analysis, absence of use of fingers, mechanic transport of psychological experiments in the teaching.

First of all, the emergence of the complicated knowledge of children about the resolution of problems depends on the rich learning experiences, which they acquired from the teaching program. The approach method of number with emphasis on the number analysis that we applied in the specific kindergartens came in accordance with the preexistent knowledge of children. All preexistent knowledge of children is useful for the later conceptual development. According to Resnick, these are pre-quantitative logic schemes and constitute the foundation of mathematic development of children (Resnick, 1983).

Consequently, the familiarization of infants with analysis activities encouraged the development of their abilities to resolve narrative problems of addition and subtraction. The results of this research show that the children are capable of applying the knowledge of analysis to the addition and subtraction. These relations recommend an important foundation that stresses a lot of mathematic concepts that young children develop (Kosyvas, 2009a).

In our approach we considered that the progressive mathematization should be connected with the mathematics of school. The acquisition of various experiences with the analyses of numbers is judged essential. The use of each number in a lot of different cases contributes to the conquest of the number concept. With the present work we propose a method of approach towards numerical notions in preschool education kindergarten, which is based on the analysis of the number itself. A set of instructive activities that aimed at the numerical development of children systematic was implemented (Kosyvas, 2010a).

The high performances of the infants, who have been initiated in the world of numbers with this method, reveal its effectiveness. However, we cannot conclude that the quality of the number approach is better, judging only from the difference of learning performances. This is one aspect of the problem. The establishment of this approach and the content of the set of instructive activities are more important (Kosyvas, 2001 – Kosyvas, 2009b).

According to the findings of this research we can conclude that the rich network of ideas that are connected with the numbers maintains and simultaneously exceeds the achievements of counting. While the children are absorbed in activities of counting, they don’t think that the counted collections may be composed by two composite parts. They also do not think of the cardinalities of these parts. The long-lasting and exclusive adherence in the counting activities may limit the perspective of emersion of new fruitful ideas. The flexible analysis and recombining of numbers help the use of reasoning strategies and work against memorization. As the children form progressively new quantitative structures and patterns, as they invent and build more fine and differentiated relations between the numbers, they become creators that go deep into in the comprehension and develop further their mathematic abilities.

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