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 (

) spline

,

$[x_{j-2}, x_{j+2}]$.

: $[x\ , x_n]$ o

( ) spline $S_{|[x\ ,x_n]}$.

,

, S $C^2[x\ , x_n]$,

spline x x_n

.

:

1) $[x\ , x_n]$

$[x\ , x_1]$.

: $S(x_0)=0=S(x_1)$

$S'(x_0)=0=S'(x_1)$, $S''(x_0)=0=S''(x_1)$, S

: $S(x)= (x-x_0)^3 (x-x_1)^3$,

.

2) $[x\ , x_n]$

,

$[x\ , x_2]$.

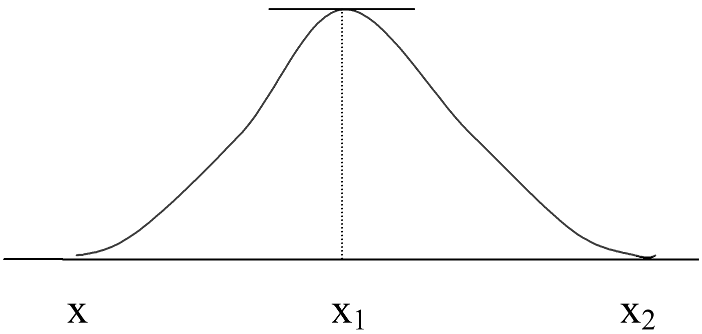
S

$[x\ ,\ x_1]$,

, :

$S_{|[x\ ,x_1]}\ (x)=\ \ (x-\ x_0)^3$

$[x_1\ ,$



$x_2]$,

:

$S_{|[x_1,x_2]}(x)=\ \ (x-\ x_2)^3$

$S\ \ C^2[x\ ,\ x_2]$

$S_{|[x\ ,x_1]}(x_1)=S_{|[x_1,x_2]}(x_1)$□

$(x_1-\ x_0)^3=\ \ (x_1-\ x_2)^3$ □ $=\ \dfrac{(x_1-\ x_2)^3}{(x_1-\ x_0)^3}$ (1)

$\lim_{x\to x_1^-} S'_{|[x\ ,x_1]}(x)=\lim_{x\to x_1^+} S'_{|[x_1,x_2]}(x)$

$3\ \ (x_1-\ x_0)^2=3\ \ (x_1-\ x_2)^2$ □ $=\ \dfrac{(x_1-\ x_2)^2}{(x_1-\ x_0)^2}$ (2)

x_0□x_1□x_2 (1) (2)

(

) spline.

(1) (2) $=\ =0$,

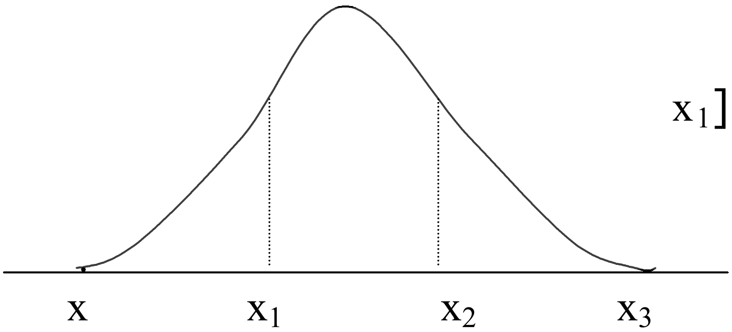
.

……………………………………………..

3) [x , x_n]

,

[x , x_3].



[x ,

x_1] [x_2 , x_3],

spline

: $S_{|[x\ ,x_1]}(x)=\ (x-x_0)^3$ $S_{|[x_2,x_3]}(x)=\ (x-x_3)^3$.

[x_1,x_2], : $S_{|[x_1,x_2]}(x)=\ x^3+\ x^2+\ x+\ $.

S C^2[x ,x_3]

x_1 x_2.

$S_{|[x\ ,x_1]}(x_1)= S_{|[x_1,x_2]}(x_1)$ $S_{|[x_2,x_3]}(x_2)= S_{|[x_1,x_2]}(x_2)$

$$\left.\begin{aligned}(x_1-x_0)^3&=\ (x_1)^3+\ (x_1)^2+\ x_1+\\(x_2-x_3)^3&=\ (x_2)^3+\ (x_2)^2+\ x_2+\end{aligned}\right\}\quad(3)$$

$$\lim_{x\to x_1^-} S'_{|[x\ ,x_1]}(x)=\lim_{x\to x_1^+} S'_{|[x_1,x_2]}(x)$$

$$\lim_{x\to x_1^-} S'_{|[x_2,x_3]}(x)=\lim_{x\to x_1^+} S'_{|[x_1,x_2]}(x)$$

$$:\ \left.\begin{aligned}3\ (x_1-x_0)^2&= 3\ (x_1)^2+2\ x_1+\\3\ (x_2-x_3)^2&= 3\ (x_2)^2+2\ x_2+\end{aligned}\right\}\quad(4)$$

……………………………………………..

$x_0 \square x_1 \square x_2 \square x_3$, (3) (4)

= = = = = = =0

spline.

4) $[x \ ,x_n]$

$[x \ ,x_4]$ (

$[x_j,x_{j+2}]$).

16 15

x_i, $0 \le i \le 4$ S´

S″ x_i, $0 \le i \le 3$

x_0 x_4.

16 15

x_i, $0 \le i \le n$

S´ S″

x_i, $0 \le i \le n-1$

$x_0=a$

$x_n=b$,

$C^2[\ ,b]$ ($=x_0,x_n=b$)

$[x_{j-2}\ ,\ x_{j+2}]$. (

.)

:=

$a= x_0 < x_1 < \ldots < x_n=b$ [a,b] $h=(b-a)/n$,

( ) spline

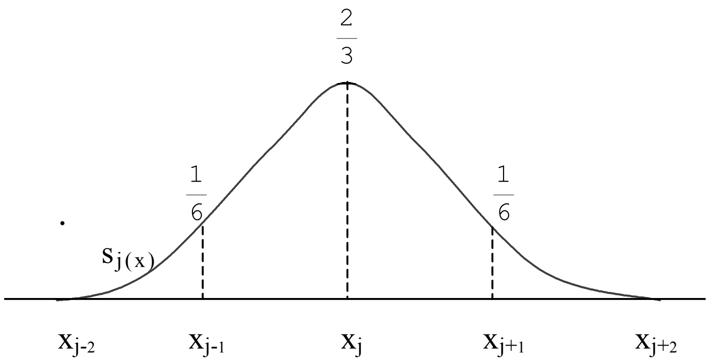
B-splines.

h $h=x_{j+1}-x_j$

B-spline s_j x_j $(2\leq j\leq n-2,)$

:

$$
s_j(x)=\begin{cases}
\dfrac{(x-x_{j-2})^3}{6h^3}, & x_{j-2}\leq x\leq x_{j-1}\\[2mm]
\dfrac{2}{3}-\dfrac{(x-x_j)^2}{h^2}-\dfrac{(x-x_j)^3}{2h^3}, & x_{j-1}\leq x\leq x_j\\[2mm]
\dfrac{1}{6}+\dfrac{x_{j+1}-x}{2h}+\dfrac{(x_{j+1}-x)^2}{2h^2}-\dfrac{(x_{j+1}-x)^3}{2h^3}, & x_j\leq x\leq x_{j+1}\\[2mm]
\dfrac{(x_{j+2}-x)^3}{6h^3}, & x_{j+1}\leq x\leq x_{j+2}\\[2mm]
0, & x\notin[x_{j-2},x_{j+2}]
\end{cases}
$$



 $\{\ _j\}$, $-1\le j\le n+1$,

, $_j := s_j|[a,b] :=$

s_j [a,b], $S_3(\ _h)$

splines

:= $a= x_0 < x_1 < \ldots < x_n=b$ [a,b] $h=(b-a)/n$, s_j B-spline x_j $(2\le j\le n-2,)$

:

$$s_j(x)=\begin{cases} \dfrac{(x-x_{j-2})^3}{6h^3}, & x_{j-2}\le x\le x_{j-1} \\ \dfrac{2}{3}-\dfrac{(x-x_j)^2}{h^2}-\dfrac{(x-x_j)^3}{2h^3}, & x_{j-1}\le x\le x_j \\ \dfrac{1}{6}+\dfrac{x_{j+1}-x}{2h}+\dfrac{(x_{j+1}-x)^2}{2h^2}-\dfrac{(x_{j+1}-x)^3}{2h^3}, & x_j\le x\le x_{j+1} \\ \dfrac{(x_{j+2}-x)^3}{6h^3}, & x_{j+1}\le x\le x_{j+2} \\ 0, & x\notin[x_{j-2}, x_{j+2}] \end{cases}$$

:

$x_{-1}=a-h$ $x_{n+1}=b+h$,

n+3 -1, 0, 1,…, n-1, n, n+1 , $_j := s_j|[a,b] :=$

s_j [a,b],.

-1, 0, 1,…, n-1, n, n+1,

C_j,

$j=-1,\ldots,n+1$:

…………………………………………..

$$\sum_{j=-1}^{n+1} C_j \quad_j = 0 \;\Rightarrow\; \sum_{j=-1}^{n+1} C_j \quad_j (x_k) = 0, \quad k=-1,\dots,n+1$$

$\Rightarrow c_{-1} \;\; _{-1}(x\;)+ c_0 \;\; _0(x\;)+ c_1 \;\; _1(x\;)+\dots+c_n \;\; _n(x\;)+ c_{n+1} \;\; _{n+1}(x\;)=0,\; k=-1,\dots,n+1$ (1)

x_{-1}, $x_0,\dots,x_n,x_{n+1}$, (1) (n+3) x (n+3):

$$\begin{cases} c_{-1} \;\; _{-1}(x_{-1})+ c_0 \;\; _0(x_{-1})+ c_1 \;\; _1(x_{-1})+ \dots +c_n \;\; _n(x_{-1})+ c_{n+1} \;\; _{n+1}(x_{-1})=0 \\ c_{-1} \;\; _{-1}(x_0)+ c_0 \;\; _0(x_0)+ c_1 \;\; _1(x_0)+ \dots +c_n \;\; _n(x_0)+ c_{n+1} \;\; _{n+1}(x_0)=0 \\ c_{-1} \;\; _{-1}(x_1)+ c_0 \;\; _0(x_1)+ c_1 \;\; _1(x_1)+ \dots +c_n \;\; _n(x_1)+ c_{n+1} \;\; _{n+1}(x_1)=0 \\ \dots\dots\dots\dots\dots\dots\dots\dots \\ c_{-1} \;\; _{-1}(x_{n-1})+ c_0 \;\; _0(x_{n-1})+ c_1 \;\; _1(x_{n-1})+\dots +c_n \;\; _n(x_{n-1})+ c_{n+1} \;\; _{n+1}(x_{n-1})=0 \\ c_{-1} \;\; _{-1}(x_n)+ c_0 \;\; _0(x_n)+ c_1 \;\; _1(x_n) + \dots +c_n \;\; _n(x_n) + c_{n+1} \;\; _{n+1}(x_n) =0 \\ c_{-1} \;\; _{-1}(x_{n+1})+ c_0 \;\; _0(x_{n+1})+ c_1 \;\; _1(x_{n+1}) \dots +c_n \;\; _n(x_{n+1})+ c_{n+1} \;\; _{n+1}(x_{n+1})=0 \end{cases}$$

,

-1， 0，···， n， n+1 x_{-1}, $x_0,\dots,x_n,x_{n+1}$:

$$\begin{cases} \frac{2}{3} c_{-1}+ \frac{1}{6} c_0 = 0 \\ \frac{1}{6} c_{-1}+ \frac{2}{3} c_0+ \frac{1}{6} c_1 = 0 \\ \frac{1}{6} c_0 + \frac{2}{3} c_1+ \frac{1}{6} c_2 = 0 \\ \dots\dots\dots\dots\dots\dots\dots\dots \\ \frac{1}{6} c_{n-2} + \frac{2}{3} c_{n-1} + \frac{1}{6} c_n = 0 \\ \frac{1}{6} c_{n-1} + \frac{2}{3} c_n + \frac{1}{6} c_{n+1}=0 \\ \frac{1}{6} c_n + \frac{2}{3} c_{n+1}=0 \end{cases}$$

……………………………………………..

: $C =$, :

$$= \begin{bmatrix} \frac{2}{3} & \frac{1}{6} & 0 & 0 & 0 \dots 0 & 0 & 0 & 0 & 0 \\ \frac{1}{6} & \frac{2}{3} & \frac{1}{6} & 0 & 0 \dots 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{1}{6} & \frac{2}{3} & \frac{1}{6} & 0 \dots 0 & 0 & 0 & 0 & 0 \\ \dots & & & & & & & & \\ 0 & 0 & 0 & 0 & 0 \dots 0 & \frac{1}{6} & \frac{2}{3} & \frac{1}{6} & 0 \\ 0 & 0 & 0 & 0 & 0 \dots 0 & 0 & \frac{1}{6} & \frac{2}{3} & \frac{1}{6} \\ 0 & 0 & 0 & 0 & 0 \dots 0 & 0 & 0 & \frac{1}{6} & \frac{2}{3} \end{bmatrix} = \frac{1}{6} \begin{bmatrix} 4 & 1 & 0 & 0 & 0 \dots 0 & 0 & 0 & 0 & 0 \\ 1 & 4 & 1 & 0 & 0 \dots 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 4 & 1 & 0 \dots 0 & 0 & 0 & 0 & 0 \\ \dots & & & & & & & & \\ 0 & 0 & 0 & 0 & 0 \dots 0 & 1 & 4 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \dots 0 & 0 & 1 & 4 & 1 \\ 0 & 0 & 0 & 0 & 0 \dots 0 & 0 & 0 & 1 & 4 \end{bmatrix} = \frac{1}{6} B$$

, - $(n+3)x(n+3)$,

$$C = \begin{bmatrix} c_{-1} \\ c_0 \\ c_1 \\ \dots \\ c_{n-1} \\ c_n \\ c_{n+1} \end{bmatrix} \quad (n+3)x1 \qquad = \begin{bmatrix} 0 \\ 0 \\ 0 \\ \dots \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$(n+3)x1$.

$= \frac{1}{6}$ $\det = (\frac{1}{6})^{n+3} \cdot \det$ (1) .

, $=[\ _{ij}]$ $i,j=1,\dots,n+3$

1 $-\frac{1}{4}$ $(= -\frac{1}{b_{11}})$

$-\frac{1}{b_{kk}}$

+1 (=2,…,n+2)

$'=[\ '_{ij}]$

$'_{11}=4$, $'_{22}=-\frac{1}{b_{11}}+4$, $'_{33}=-\frac{1}{b_{22}}+4$,…,

$'_{(\ +1)(\ +1)}=-\frac{1}{b_{kk}}+4$,…, $'_{(n+3)(n+3)}=-\frac{1}{b_{n+2\ \ 2}}+4$.

$'=[\ '_{ij}]$

(i,j=1,…,n+3) det = det ′ (2).

$'=[\ '_{ij}]$

: $3< \ '_{mm}\square 4$, m=1,2,…,n+3.

:

$_1$ (m=1): $3< \ '_{11}\square 4$ $\square$ $3<4\square 4$

(m=): $3< \ ' \ \square 4$ (3) .

$_{+1}$ (m= +1): $3< \ '_{(\ +1)(\ +1)}\square 4$:

$(3)\ \square\ \frac{1}{4}\square\frac{1}{b_{kk}}<\frac{1}{3}\ \square\ -\frac{1}{3}<-\frac{1}{b_{kk}}\square-\frac{1}{4}\ \square$

$3<-\frac{1}{3}+4<-\frac{1}{b_{kk}}+4\square-\frac{1}{4}+4\square 4\ \square\ 3< \ '_{(\ +1)(\ +1)}\square 4$

$3< \ '_{mm}\square 4$, m

m=1,2,…,n+3 :

$$3 < \ '_{11} \square 4$$

$$3 < \ '_{22} \square 4$$

$$3 < \ '_{33} \square 4$$

…………

$$3< \ '_{(n+3)(n+3)}\square 4$$

:

$$3^{n+3} < \ '_{11}\cdot \ '_{22}\cdot \ '_{33}\cdot \ldots \cdot \ '_{(n+3)(n+3)} \square \ 4^{n+3}$$

′=[′ij] :

3^{n+3}<det ′ □4^{n+3}

(2) 3^{n+3}<det □4^{n+3}

$(\frac{1}{6})^{n+3}\cdot 3^{n+3}<(\frac{1}{6})^{n+3}\cdot$det □$(\frac{1}{6})^{n+3}\cdot 4^{n+3}$

(1) $(\frac{1}{2})^{n+3}$<det □$(\frac{2}{3})^{n+3}$

detA 0,

C=

$C_j=0$, j j= -1,…,n+1.

$_j$ S_3( ) □ n+3

{ $_j$}, -1≤j≤n+1

S_3( )

(n+3)□ t S_3($_h$) (

t S_3( )) $_{-1}$, $_0$, $_1$, …, $_{n-1}$, $_n$, $_{n+1}$,t

. C_i,

i=-1,…,n+1 $t=\sum_{i=-1}^{n+1} C_i\ _i.$

{ $_j$}, -1≤j≤n+1

S_3($_h$) splines

h.

 k,n ☐ ƒ,

ƒ(x)☐cos(k x), x [0,1],

[0,1],

x_i☐ih, i=0,1,2,…,n, h=1/n.

a) ☐ $☐_n$ Lagrange

ƒ x_i.

$$☐_☐ ☐ \max_{0 £ x £ 1} |ƒ(x) - ☐(x)|$$

k n

k n,

$☐_☐$ 0 k ☐ n ☐ .

b) s spline [0,1]

ƒ x_i.

$$☐_s ☐ \max_{0 £ x £ 1} |ƒ(x) - s(x)|$$

k n

k n,

$☐_s$ 0 k ☐ n ☐ .

……………………………………………..

:

a)

$f(x) = \cos(k\ \ x),\ \ x\ \ [0,1],$

n+1 $(f\ \ C^{n+1}[0,1])$, ’

$[0,1]$

,

$$\| f - \ _n \|_{\ } \le \frac{\ ^{n+1}}{4\ (n+1)} \| f^{(n+1)} \|_{\ } \qquad (1)$$

:

$$\max_{0 \le x \le 1} | f(x) - \ (x) | \le \frac{\ ^{n+1}}{4\ (n+1)} \max_{0 \le x \le 1} | f^{(n+1)}(x) | \qquad (2)$$

$$|f^{(n+1)}(x)| = \begin{cases} (k\ \ )^{n+1} \sin(k\ \ x), & n=2 \\ (k\ \ )^{n+1} \cos(k\ \ x), & n=2\ \ +1 \end{cases}$$

: $\max_{0 \le x \le 1} |f^{(n+1)}(x)| = (k\ \ )^{n+1}$.

(2) : $\ _{\ } = \max_{0 \le x \le 1} | f(x) -$

$\ (x) | \le$

$$\le \frac{\ ^{n+1}}{4\ (n+1)} (k\ \ )^{n+1} = \frac{\ ^{n+1}}{4\ (n+1)} = \frac{kp}{4\ (n+1)\ ^{n+1}} = \frac{1}{4\ (n+1)} \cdot \left(\frac{kp}{n}\right)^{n+1}$$

$\ _{\ }$ k n

:

$$\frac{1}{4\ (n+1)} \cdot \left(\frac{kp}{n}\right)^{n+1} \qquad (3)$$

……………………………………………..

$\frac{1}{4\,(n+1)} \cdot \left(\frac{k\pi}{n}\right)^{n+1}$

k □ n □ n>k .

b)

$|f''(x)|=(k\ )^2\ \cos(k\ x),$

:

$\Delta_s = \max_{0 \le x \le 1} |f(x) - s(x)| \le \frac{h^2}{8} \max_{0 \le x \le 1} |f''(x)| =$

$= \frac{\left(\frac{1}{n}\right)^2}{8} \cdot (k\pi)^2 = \frac{(k\pi)^2}{8n^2} = \frac{1}{8}\left(\frac{k\pi}{n}\right)^2$

Δ_s k n

:

$\frac{1}{8}\left(\frac{k\pi}{n}\right)^2$, (4)

$\frac{1}{8}\left(\frac{k\pi}{n}\right)^2$

k □ n □ n□k^2.

……………………………………………..

□

$$f(x) □ \begin{cases} 0, & 0 \le x \le 1, \\ (x-1)^4, & 1 < x \le 2. \end{cases}$$

a) f $[0,2]$

□

$$□(x) □ \begin{cases} 0, & 0 \le x \le 1, \\ \quad + \quad (x-1) + \quad (x-1)^2 + \quad (x-1)^3 & 1 < x \le 2. \end{cases}$$

, , , ,

□ $C^1[0,2]$ □ $(0) □ f(0)$, $□'(0) □ f'(0)$,

$□(1) □ f(1)$, □ $(2) □ f(2)$, $□'(2) □ f'(2)$.

b) □

spline s f $\{0,1,2\}$

$s'(0) □ f'(0)$, $s'(2) □ f'(2)$;

: □ $C^1[0,2]$, □

1 : lim P = P x ® 1- x ® 1+

lim P = p & lim P = a x ® 1- x ® 1+

: $= 0$

$□'(x) □ \quad +2 \quad x-2 \quad +3 \quad x^2-6 \quad x+3$

$□'(1^+) □ \quad +2 \quad -2 \quad +3 \quad ^2-6 \quad +3 \quad =$

……………………………………………..

□′(1⁻)□ 0

□ $C^1[0,2]$ □′(1⁺)□ □′(1⁻)

: = 0

□(2)□ + (2-1) + $(2-1)^2$+ $(2-1)^3$= +

$f(2)=(2-1)^4=1$ + =1 (1)

$1< x \le 2$ $f(x)$□ $(x-1)^4= x^4-4x^3+6x^2-4x+1$

$f'(x)$□ $4x^3-12x^2+12x-4$

$f'(2)= 4 2^3-12\ 2^2+12\ 2-4= 4$

$1< x \le 2$ □′(x)= 2 x-2 +3 x^2-

6 x+3

□′(2)=2 2-2 +3 2^2-6 2+3

=4 -2 +12 -12 +3

= 2 +3

□′(2)□$f'(2)$ 2 +3 = 4 (2)

(1) + = 1 ; 2 +3 =4 □ -2 -2 =-2 ; 2 +3 = 4 □ (2)

+ = 1 ; =2 □ +2=1 ; =2 □ =-1 ; = 2

=0, =0, =-1 =2

.

□ :

………………………………………..

$$\square(x)\square\begin{cases} 0, & 0\leq x \leq 1,\\ -(x-1)^2+ 2(x-1)^3\ , & 1< x \leq 2.\end{cases}$$

:

$$\square(x)\square\begin{cases} 0, & 0\leq x \leq 1,\\ 3x^3 -7x^2+ 8x-3, & 1< x \leq 2.\end{cases}$$

b) :

□

spline s *ƒ* □ $C^1[0,2]$

1

:

$0\leq x < 1$ $\square''(x)\square 0$ $\square''(1^-)\square 0$

$1< x \leq 2$ $\square''(x)\square 12x-14$ $\square''(1^+)\square 12-14=-2$ $\square''(1^-)\ \square\square''(1^+)$.

□

spline s *ƒ*

$\{0,1,2\}$.

□ f $C^2[a,b]$ s

spline f

x_i $a= x_0 < x_1 < \ldots < x_n=b$ $[a,b]$

$s'(x_0)=f'(x_0)$, $s'(x_n)=f'(x_n)$.

$\int_a^b$ [illegible],

$\int_a^b$ [illegible] $\int_a^b$ [illegible].

:

$\int_a^b$ [illegible] $\sum_{i=0}^{n-1} \int_{x_i}^{x_{i+1}}$ [illegible]

( )

$= \sum_{i=0}^{n-1}$ [illegible] $\Big|_{x_i}^{x_{i+1}}$ $\sum_{i=0}^{n-1} \int_{x_i}^{x_{i+1}}$ [illegible]

……………………………………………..

f ', S' S" :

$$\sum_{i=0}^{n-1} f' \text{[illegible]} \Big|_{x_i}^{x_{i+1}} \text{[illegible]} 0$$

S_n''' [illegible] se 3 (S_n''' [illegible] $=C_m$) :

$$\int_{x_i}^{x_{i+1}} \text{[illegible]} = 0$$

:

$(f'' - s'')^2 = f''^2 - 2 f'' s'' + s''^2$

$\Rightarrow f''^2 = (f''-s'')^2 + 2f''s'' - s''^2 = (f''-s'')^2 + 2f''s'' - 2s''^2 + s''^2$

$\Rightarrow f''^2 = (f''-s'')^2 + 2(f''-s'')s'' + s''^2$

:

$$\int_a^b f''^2 \text{[illegible]}$$

$$\int_a^b f'' \text{[illegible]} 2 \int_a^b f'' \text{[illegible]} \int_a^b s'' \text{[illegible]}$$

$$\int_a^b f'' \text{[illegible]} \int_a^b s'' \text{[illegible]} \int_a^b s'' \text{[illegible]}.$$

$f(x)=x^5+x^4$, {-2,-1,0,1,2}

[-2,2]. ,

, spline,

f -2,-1,0,1,2.

:

:

$$s(x)=\frac{1}{6h}\left[s_i''(x-x_{i-1})^3-s_{i-1}''(x-x_i)^3\right]+\left(\frac{y_i}{h}-s_i''\frac{h}{6}\right)(x-x_{i-1})$$

$$-\left(\frac{y_{i-1}}{h}-s_{i-1}''\frac{h}{6}\right)(x-x_i),\qquad x\in[x_{i-1},x_i].\qquad(1)$$

$$s_{i-1}''+4s_i''+s_{i+1}''=\frac{6}{h^2}(y_{i-1}-2y_i+y_{i+1}),\ i=1,\dots,n-1\qquad(2)$$

spline (1)

:

$x\in[-2,-1]$ $\quad s(x)=\frac{1}{6}\left[s''(-1)(x-(-2))^3-s''(-2)(x-(-1))^3\right]+$

$$+\left(\frac{f(-1)}{1}-s''(-1)\frac{1}{6}\right)(x-(-2))-\left(\frac{f(-2)}{1}-s''(-2)\frac{1}{6}\right)(x-(-1)),$$

$x\in[-1,0]$ $\quad s(x)=\frac{1}{6}\left[s''(0)(x-(-1))^3-s''(-1)(x-0)^3\right]+$

$$+\left(\frac{f(0)}{1}-s''(0)\frac{1}{6}\right)(x-(-1))-\left(\frac{f(-1)}{1}-s''(-1)\frac{1}{6}\right)(x-0),$$

$x \in [0, 1]$ $\quad s(x) = \frac{1}{6}[s''(1)(x-0)^3 - s''(0)(x-1)^3] +$

$$+\left(\frac{f(1)}{1} - s''(1)\frac{1}{6}\right)(x-0) - \left(\frac{f(0)}{1} - s''(0)\frac{1}{6}\right)(x-1),$$

$x \in [1, 2]$ $\quad s(x) = \frac{1}{6}[s''(2)(x-1)^3 - s''(1)(x-2)^3] +$

$$+\left(\frac{f(2)}{1} - s''(2)\frac{1}{6}\right)(x-1) - \left(\frac{f(1)}{1} - s''(1)\frac{1}{6}\right)(x-2).$$

$s''(-2)$, $s''(-1)$, $s''(0)$, $s''(1)$ $s''(2)$

(2), :

$$\left.\begin{aligned} s''(-2) + 4s''(-1) + s''(0) &= \frac{6}{1^2}[f(-2) - 2f(-1) + f(0)] \\ s''(-1) + 4s''(0) + s''(1) &= \frac{6}{1^2}[f(-1) - 2f(0) + f(1)] \\ s''(0) + 4s''(1) + s''(2) &= \frac{6}{1^2}[f(0) - 2f(1) + f(2)] \end{aligned}\right\} \quad (3)$$

:

$f(-2) = (-2)^5 + (-2)^4 = -32 + 16 = -16$

$f(-1) = (-1)^5 + (-1)^4 = -1 + 1 = 0$

$f(0) = 0^5 + 0^4 = 0$

$f(1) = 1^5 + 1^4 = 2$

……………………………………………..

$f(2) = 2^5+2^4=32+16=48$,

(3) :

$$\left.\begin{array}{l} s''(-2)+ 4 s''(-1) + s''(0) = -96 \\ s''(-1) +4 s''(0) + s''(1) = 12 \\ s''(0) + 4 s''(1) + s''(2) = 264 \end{array}\right\}$$

spline

$s''(-2) = 0 = s''(2)$:

$$\left.\begin{array}{l} 4 s''(-1) + s''(0) = -96 \\ s''(-1) +4 s''(0) + s''(1) = 12 \\ s''(0) + 4 s''(1) = 264 \end{array}\right\}$$

: $s''(-1)=$, $s''(0)=$, $s''(1)=$

$$\left.\begin{array}{l} : 4 \quad + \quad = -96 \\ +4 \quad + \quad = 12 \\ + 4 \quad = 264 \end{array}\right\} \quad (4)$$

:

$$D=\begin{vmatrix} 4 & 1 & 0 \\ 1 & 4 & 1 \\ 0 & 1 & 4 \end{vmatrix} = 56, \qquad D = \begin{vmatrix} -96 & 1 & 0 \\ +12 & 4 & 1 \\ 264 & 1 & 0 \end{vmatrix} =-1224$$

…………………………………………..

$$D = \begin{vmatrix} 4 & -96 & 0 \\ 1 & +12 & 1 \\ 0 & 264 & 4 \end{vmatrix} =-480 \qquad D = \begin{vmatrix} 4 & 1 & -96 \\ 1 & 4 & +12 \\ 0 & 1 & 264 \end{vmatrix} =3816$$

$$= \frac{D_a}{D} = \frac{-1224}{56} = \frac{-153}{7}$$

$$= \frac{D_b}{D} = \frac{-480}{56} = \frac{-60}{7}$$

$$= \frac{D_g}{D} = \frac{3816}{56} = \frac{477}{7}$$

: $s''(-1)= \frac{-153}{7}$, $s''(0)= \frac{-60}{7}$, $s''(1)= \frac{477}{7}$

:

x e[-2, -1]

$$s(x) = \frac{1}{6}[(\frac{-153}{7})(x-(-2))^3-s''(-2)(x-(-1))^3]+(0-(\frac{-153}{7})\frac{1}{6})(x-(-2))-$$

$$-(-16-0.\frac{1}{6})(x-(-1)) = \frac{-51}{14}[(x+2)^3-(x+2)]+16(x+1),$$

x e[-1, 0]

$$s(x)= \frac{1}{6}[\frac{-60}{7}(x-(-1))^3-(\frac{-153}{7})(x-0)^3]+(0-(\frac{-60}{7})\frac{1}{6})(x-(-1))-$$

$$-(0-(\frac{-153}{7})\frac{1}{6})(x-0)=-\frac{1}{14}[20(x+3)^3-51x^3-20(x+1)+51x],$$

……………………………………………..

x e[0, 1]

$$s(x) = \frac{1}{6}\left[\frac{477}{7}(x-0)^3 - \left(\frac{-60}{7}\right)(x-1)^3\right] + \left(2 - \frac{477}{7}\,\frac{1}{6}\right)(x-0) -$$

$$-\left(0 - \left(\frac{-60}{7}\right)\frac{1}{6}\right)(x-1) = \frac{1}{14}\left[159x^3+20(x-1)^3-131x-20(x-1)\right]$$

x e[1, 2]

$$s(x) = \frac{1}{6}\left[0.(x-1)^3 - \frac{477}{7}(x-2)^3\right] + \left(48 - 0\,.\,\frac{1}{6}\right)(x-1) - \left(2 - \frac{477}{7}\,\frac{1}{6}\right)(x-2)$$

$$= -\frac{1}{14}\left[159(x-2)^3-672(x-1)-131(x-2)\right]$$

spline,

f -2,-1,0,1,2

:

$$S(x)=\begin{cases} \frac{\square 1}{14}\left[51(x+2)^3-51(x+2)-224(x+1)\right], & x\,e[-2,-1] \\ \frac{1}{14}\left[-20(x+3)^3+51x^3+20(x+1)-51x\right], & x\,e[-1,\ 0] \\ \frac{1}{14}\left[159x^3+20(x-1)^3-131x-20(x-1)\right], & x\,e[\ 0,\ 1] \\ \frac{\square 1}{14}\left[159(x-2)^3-672(x-1)-131(x-2)\right], & x\,e[\ 1,\ 2] \end{cases}$$