

3^ο Λυμένο Παράδειγμα: $\frac{\eta\mu x}{x}$ - Άλλες Εφαρμογές

Να υπολογιστεί το $\lim_{x \rightarrow 0} \frac{2x\eta\mu x}{1-\sigma\omega 2x}$

Λυση

$$\lim_{x \rightarrow 0} \frac{2x\eta\mu x}{2\eta\mu^2 x} = \lim_{x \rightarrow 0} \frac{x}{\eta\mu x} = \lim_{x \rightarrow 0} \frac{1}{\frac{\eta\mu x}{x}} = 1$$

$$* \quad 1 - \sigma\omega 2x = 2\eta\mu^2 x \quad *$$

4^ο Λυμένο Παράδειγμα

$$\lim_{x \rightarrow 0} \frac{\eta\mu(\eta\mu x)}{2x^2 - x}$$

Ποδιγω και διαιω με το $\eta\mu x$

$$\lim_{x \rightarrow 0} \frac{\eta\mu(\eta\mu x)}{\eta\mu x} \cdot \frac{\eta\mu x}{x(2x-1)}$$

Προσοχή:

$$\lim_{x \rightarrow 0} \frac{\eta\mu(\eta\mu x)}{\eta\mu x} \quad \text{θετω} \quad u = \eta\mu x.$$

Τότε όταν $x \rightarrow 0$ έπειτα $u \rightarrow \eta\mu 0 = 0$

$$\text{Άρα} \quad \lim_{x \rightarrow 0} \frac{\eta\mu(\eta\mu x)}{\eta\mu x} = \lim_{u \rightarrow 0} \frac{\eta\mu u}{u} = 1$$

$$\text{Επίσης} \quad \lim_{x \rightarrow 0} \frac{\eta\mu x}{x} \cdot \frac{1}{2x-1} = 1 \cdot (-1) = -1$$

$$\text{Άρα } \lim_{x \rightarrow 0} \frac{\eta\mu(\eta\mu x)}{2x^2 - 1} = 1 \cdot (-1) = -1$$

5^ο Λυμένο Παράδειγμα

$$\lim_{x \rightarrow 1} \frac{\eta\mu \pi x}{1-x^2} = ?$$

Λύση

$$\lim_{x \rightarrow 1} \frac{\eta\mu(\pi - \pi x)}{1-x^2} = \lim_{x \rightarrow 1} \frac{\eta\mu[\pi(1-x)]}{(1-x)(1+x)}$$

$$= \lim_{x \rightarrow 1} \frac{\eta\mu[\pi(1-x)]}{\pi(1-x)} \cdot \frac{\pi}{1+x}$$

$$= 1 \cdot \frac{\pi}{2} = \frac{\pi}{2}$$

6^ο Λυμένο Παράδειγμα

$$\lim_{x \rightarrow 0} \frac{\epsilon\varphi^2 x + 5x}{x + 2x^2}$$

Λύση

$$\lim_{x \rightarrow 0} \frac{\frac{\epsilon\varphi^2 x}{x} + 5}{1 + 2x} \quad (\text{διαφορούμε με } x)$$

$$= \lim_{x \rightarrow 0} \frac{\frac{\eta\mu x}{x} \cdot \frac{\eta\mu x}{\sigma\omega^2 x} + 5}{1 + 2x} = \frac{1 \cdot \frac{0}{1} + 5}{1 + 0} = 5$$

Άλλα Παραδείγματα

Π) Να υπολογιστούν τα όρια

$$\alpha) \lim_{x \rightarrow 0} \frac{\sqrt{1+\eta\mu^2 x} - \sigma\omega x}{x \eta\mu x}$$

$$\theta) \lim_{x \rightarrow 0} \frac{\epsilon\varphi x}{x}$$

$$\delta) \lim_{x \rightarrow \frac{\pi}{2}} \frac{\sigma\omega x}{\pi - 2x}$$

$$\delta) \lim_{x \rightarrow 2} \frac{\eta\mu(x^2 - 5x + 6)}{x^2 - 6x + 8}$$

$$\epsilon) \lim_{x \rightarrow 0} \frac{\eta\mu(\sigma\omega x - 1) \cdot \sigma\omega(x - \eta\mu x)}{\sigma\omega^2 x - 1}$$

$$\sigma\zeta) \lim_{x \rightarrow 0} \frac{\eta\mu 3x}{\eta\mu 6x}$$

$$\zeta) \lim_{x \rightarrow 0} \frac{\epsilon\varphi 6x}{\epsilon\varphi 3x}$$

$$\eta) \lim_{x \rightarrow 0} \frac{\eta\mu x^2}{x \cdot \eta\mu x}$$

$$\theta) \lim_{x \rightarrow 0} \frac{x}{\sqrt{2} - \sqrt{1+\sigma\omega x}}$$

$$\iota) \lim_{x \rightarrow 0} \frac{\eta\mu(\eta\mu x)}{\pi - x}$$

$$\kappa) \lim_{x \rightarrow 0} \frac{\sqrt{1+\eta\mu x} - \sqrt{1-\eta\mu x}}{x}$$

$$\lambda) \lim_{x \rightarrow 0} \frac{x^2 - 4\eta\mu x}{2x + \sigma\omega x - 1}$$

$$\mu) \lim_{x \rightarrow 0} \frac{\eta\mu x + 5x}{2\epsilon\varphi x + x}$$

$$\nu) \lim_{x \rightarrow 0} \frac{\eta\mu 3x}{x}$$

$$\xi) \lim_{x \rightarrow 0} \frac{\eta\mu 3x + 9x}{4x + \eta\mu 2x}$$

$$\omicron) \lim_{x \rightarrow 0} \frac{\eta\mu x}{\sqrt{x+4} - 2}$$

$$\pi) \lim_{x \rightarrow 0} \frac{\eta\mu x \sigma\omega x - \eta\mu x}{\sqrt{x^2 + 9} - 3}$$

$$\rho) \lim_{x \rightarrow 0} \frac{\epsilon\varphi x - \eta\mu x}{x^3}$$

$$\sigma) \lim_{x \rightarrow 0} \frac{\sigma\omega x - \sqrt{\sigma\omega x}}{\eta\mu x}$$

(2) Βρείτε, αν υπάρχει, το $\lim_{x \rightarrow 0} f(x)$

όπου

$$f(x) = \begin{cases} \frac{x^2 + \eta \mu x}{\sqrt{1+x} - \sqrt{1+x^2}} & , x > 0 \\ \frac{5\eta \mu x + 9x}{3\eta \mu x + 4x} & , x < 0 \end{cases}$$

(3) Βρείτε τα όρια

i) $\lim_{x \rightarrow 0} \frac{\eta \mu x}{x}$

(ii) $\lim_{x \rightarrow 0} \frac{1 - \sigma \omega x}{x - 3|x|}$

iii) $\lim_{x \rightarrow 0} \frac{x^2 + \eta \mu x}{\sqrt{1+x^2} - \sqrt{1+x}}$

(iv) $\lim_{x \rightarrow 2} \frac{\eta \mu(x-2)}{\sqrt{x+7} - 3}$

v) $\lim_{x \rightarrow 1} \frac{\eta \mu(x-1)}{\sqrt[3]{x} - 1}$

vi) $\lim_{x \rightarrow 1} \frac{x^2 \eta \mu(x-1)}{\sqrt{x} - 1}$

vii) $\lim_{x \rightarrow 0} \frac{\eta \mu x + \eta \mu^2 x + \dots + \eta \mu(\nu x)}{x}$

viii) $\lim_{x \rightarrow 0} \frac{3x + \epsilon \varphi x}{x + \eta \mu x}$

ix) $\lim_{x \rightarrow 0} \frac{\epsilon \gamma x + 1 - \sigma \omega^2 x}{x \eta \mu x}$

x) $\lim_{x \rightarrow 0} \frac{3\eta \mu x - x}{2\eta \mu x - x}$

xi) $\lim_{x \rightarrow 0} \frac{\sqrt{8x^2 + 4} - 2\sigma \omega x}{x^2}$