A Geometry Problem Solved Using Dot Product.

Find the angle between a diagonal of a cube and one of its edges.

Let 1 be the length of an edge and introduce a coordinate system as shown in Figure 1.

If we let $v = (0, 0, 1)$ one edge of cube, then the vector $u = (1, 1, 1)$ is a diagonal of the cube. It follows from the definition of dot product that the angle ϕ between u and the edge v satisfies

$$\cos\phi = \frac{u \cdot v}{\|u\| \cdot \|v\|} \Rightarrow$$

$$\cos\phi = \frac{1}{\sqrt{1^2 + 1^2 + 1^2} \cdot \sqrt{0^2 + 0^2 + 1^2}} \Rightarrow \cos\phi = \frac{1}{\sqrt{3}}$$

With the help of a calculator we obtain

$$\phi = \arccos(\frac{1}{\sqrt{3}}) \cong 54.74^o$$

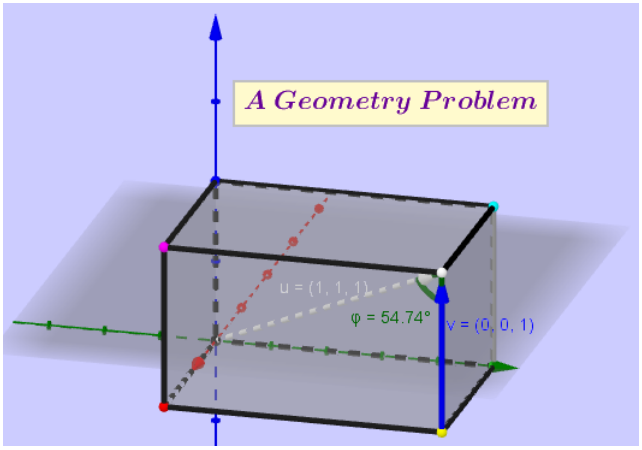

Figure 1: The angle between a diagonal of a cube and one of its edges