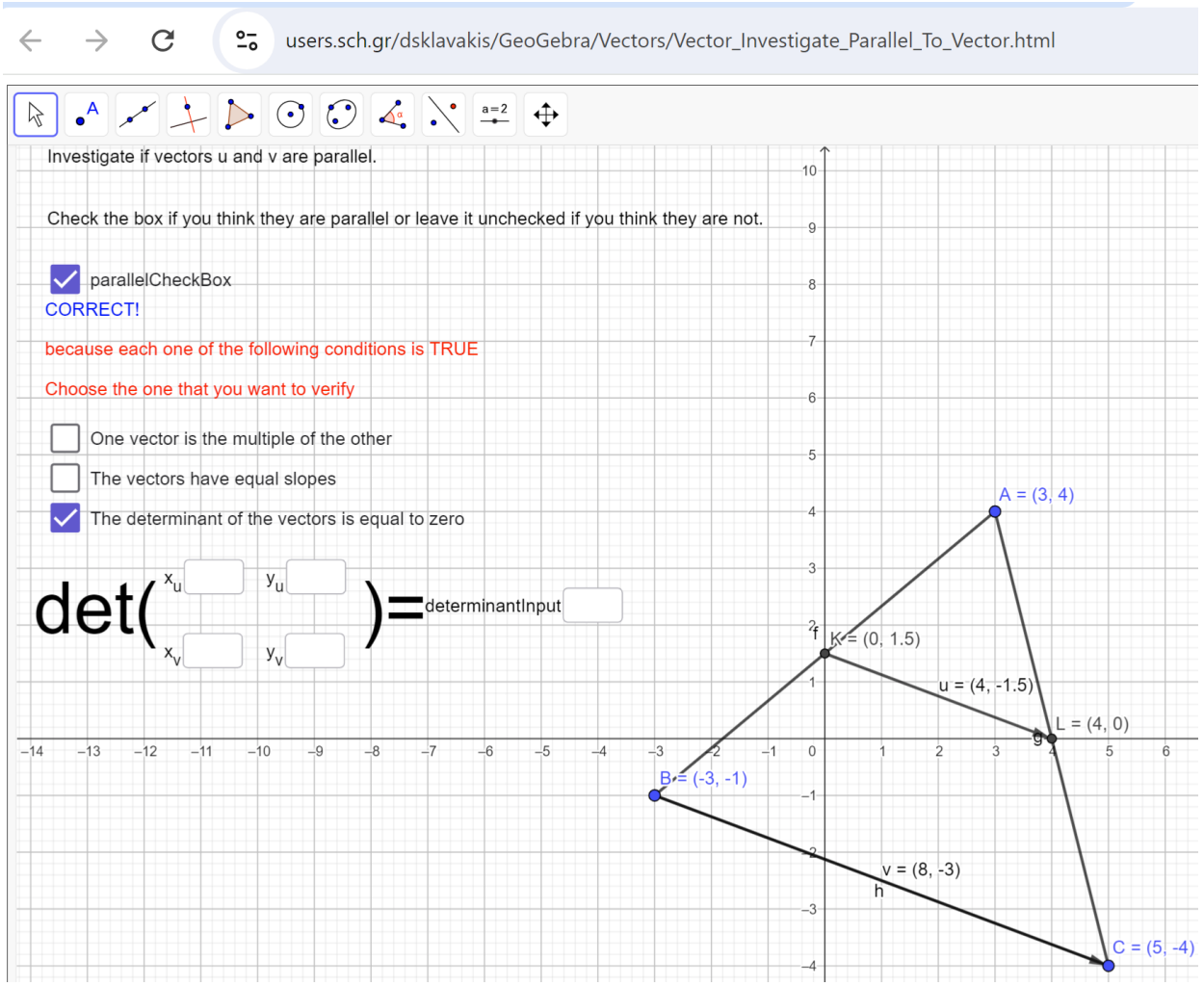
The MATHESIS Integrated Intelligent Maths Tutoring System



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Overview

- Introduction to Model-Tracing Tutors
- The GeoGebra Intelligent m-Tutors
  - The Domain Expertise Model
  - The Tutoring Expertise Model (Demo)
  - The Student Model
- The MATHESIS LMS
- Further Work
- Discussion

Bloom's 2 Sigma Problem

The Discovery

In 1984 Benjamin Bloom discovered a teaching method for drastically improving educational efficiency, by a factor of two standard deviations, 2σ (2 sigma)

The "average" student within a given class could now perform better than 49 out of every 50 students within a traditional classroom setting.

1. Mastery learning: Each student must achieve true mastery of a topic before moving on to the next, more advanced subject.

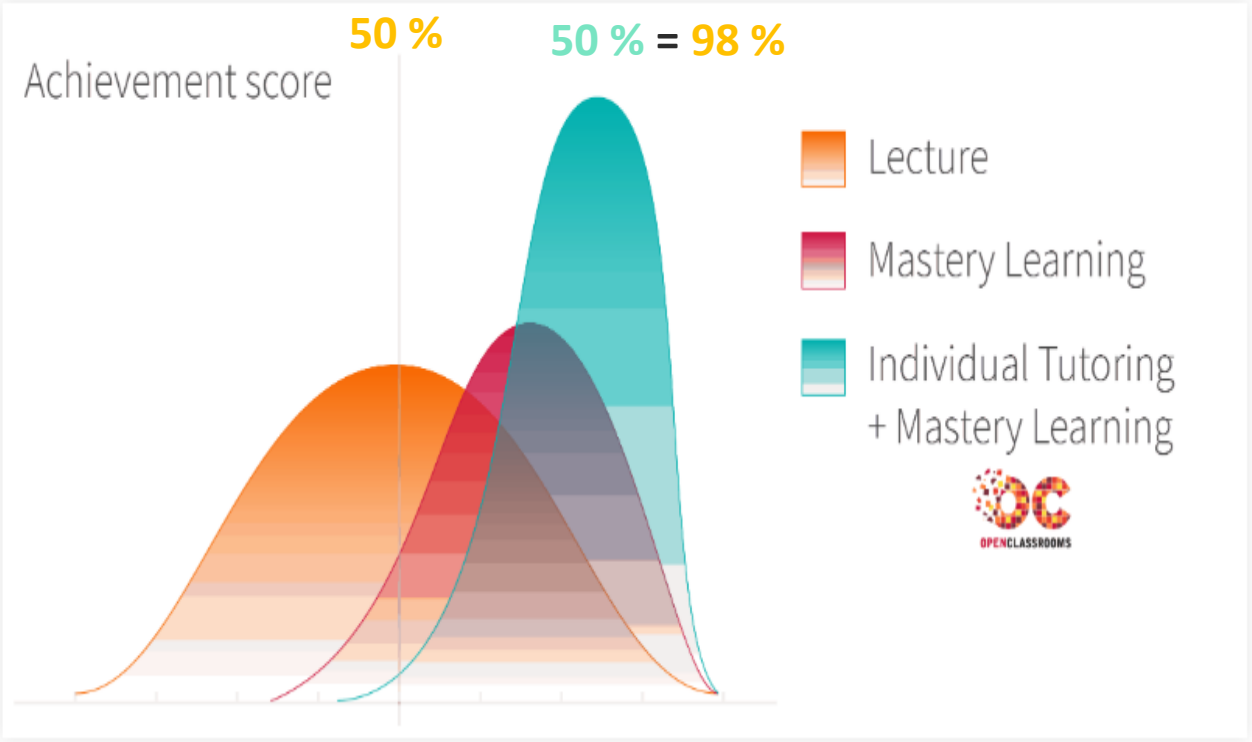
2. One-to-One tutoring: Each student is provided with a personal tutor who guides him through their learning, suggesting specific exercises and unlocking the individual student's potential on an ongoing basis and ensuring they truly "get" the subject.

Bloom's 2 Sigma Problem

The Unsolved Issue of Resources

Mastery Learning: substantial time would be needed to set up a mastery-oriented teaching framework, and the move away from hour-long teaching blocks would be hugely disruptive to the traditional learning environment.

One-to-One Tutoring: time and cost- intensive, and incredibly difficult to implement for large groups of students (scalability).



https://www.classcentral.com/report/wp-content/uploads/2016/03/Achievement-3rd.png

The problem of implementing educational environments as effective as mastery learning + individual tutoring was termed by Bloom as "the 2 Sigma problem".

Intelligent Tutoring Systems

A Solution to the 2 Sigma Problem

Intelligent Tutoring Systems (ITS), and particularly **Model-Tracing Tutors** (MTT), have been proven quite successful in various STEM disciplines (mathematics, physics, programming).Their success lies in three expertise models:

- The ***domain expertise model*** or ***problem solver***, which represents the problem-solving knowledge of the tutored domain.
- The ***pedagogical*** *or* ***tutoring model*** which represents the teaching knowledge of the system. It guides the student *within* exercises (**One-to-One tutoring**) and *between* exercises (**mastery learning**).
- The ***student model***, which represents the student's mastery level of the domain's cognitive skills (*competences*). Used by the tutoring model to control mastery learning.

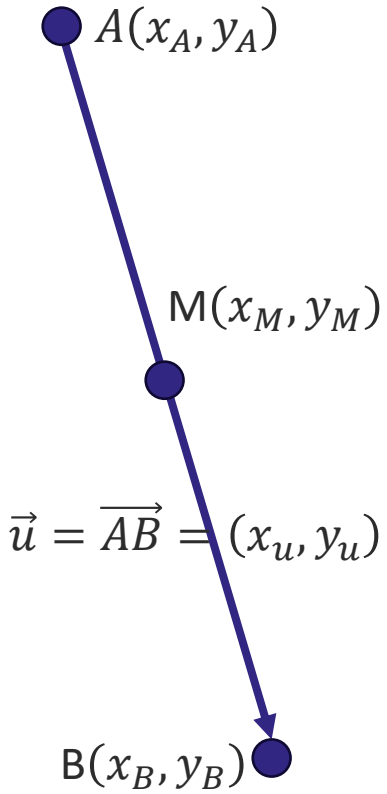
The GeoGebra Intelligent m-Tutors

The Domain Expertise Model

- Calculation of vector coordinates from its initial and terminal points: $x_u = x_B - x_A,\ \ y_u = y_B - y_A$
- Calculation of vector magnitude from its coordinates: $|\vec{u}| = \sqrt{x_u^2 + y_u^2}$
- Calculation of the distance of two points from their coordinates: $(AB) = \sqrt{(x_B - x_A)^2 + (y_B - y_A)^2}$
- Calculation of a segment's midpoint from the coordinates of its endpoints: $x_M = \frac{x_A + x_B}{2},\ \ y_M = \frac{y_A + y_B}{2}$
- Investigation of whether two vectors are parallel/vertical or not

$$\overrightarrow{u_1} = (x_1, y_1) // u_2 = (x_2, y_2) \Leftrightarrow \frac{y_1}{x_1} = \frac{y_2}{x_2} \Leftrightarrow x_1 y_2 - x_2 y_1 = 0 \Leftrightarrow \begin{vmatrix} x_1 & y_1 \\ x_2 & y_2 \end{vmatrix} = 0$$

- Calculation of a line's equation from its slope and one point: $y - y_A = \lambda(x - x_A)$
- Calculation of a line's equation from two points: $\lambda = \frac{y_B - y_A}{x_B - x_A}, y - y_A = \lambda(x - x_A)$



The GeoGebra Intelligent m-Tutors

The Tutoring Model Demo: Guiding the solution of an exercise

https://users.sch.gr/dsklavakis/GeoGebra/MATHESIS_Main_Frameset.htm

Given the triangle ABC,

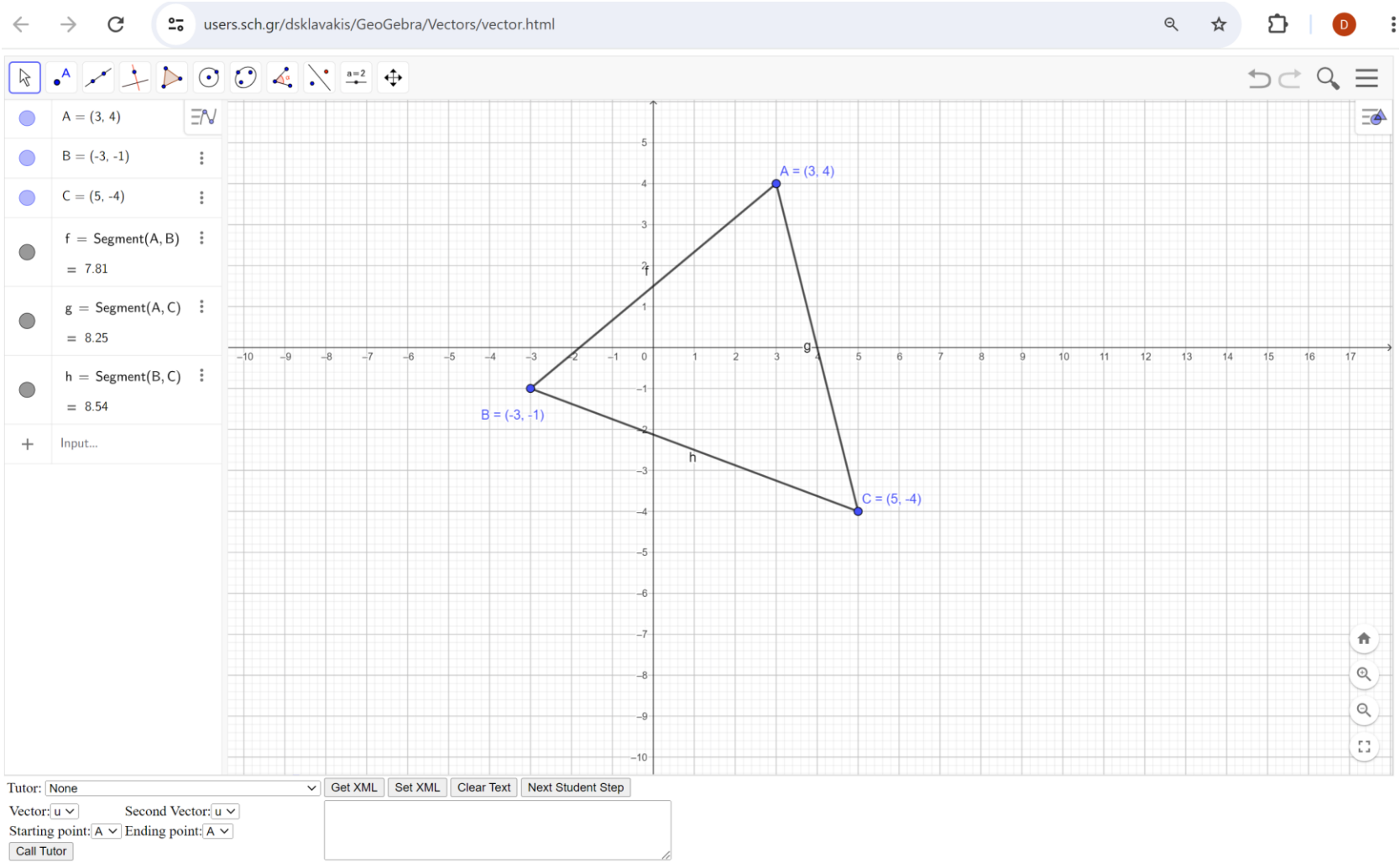
with vertices $A(3,4), B(-3,-1)$

and $C(5,-4)$.

1. Find the midpoints, D *and* E

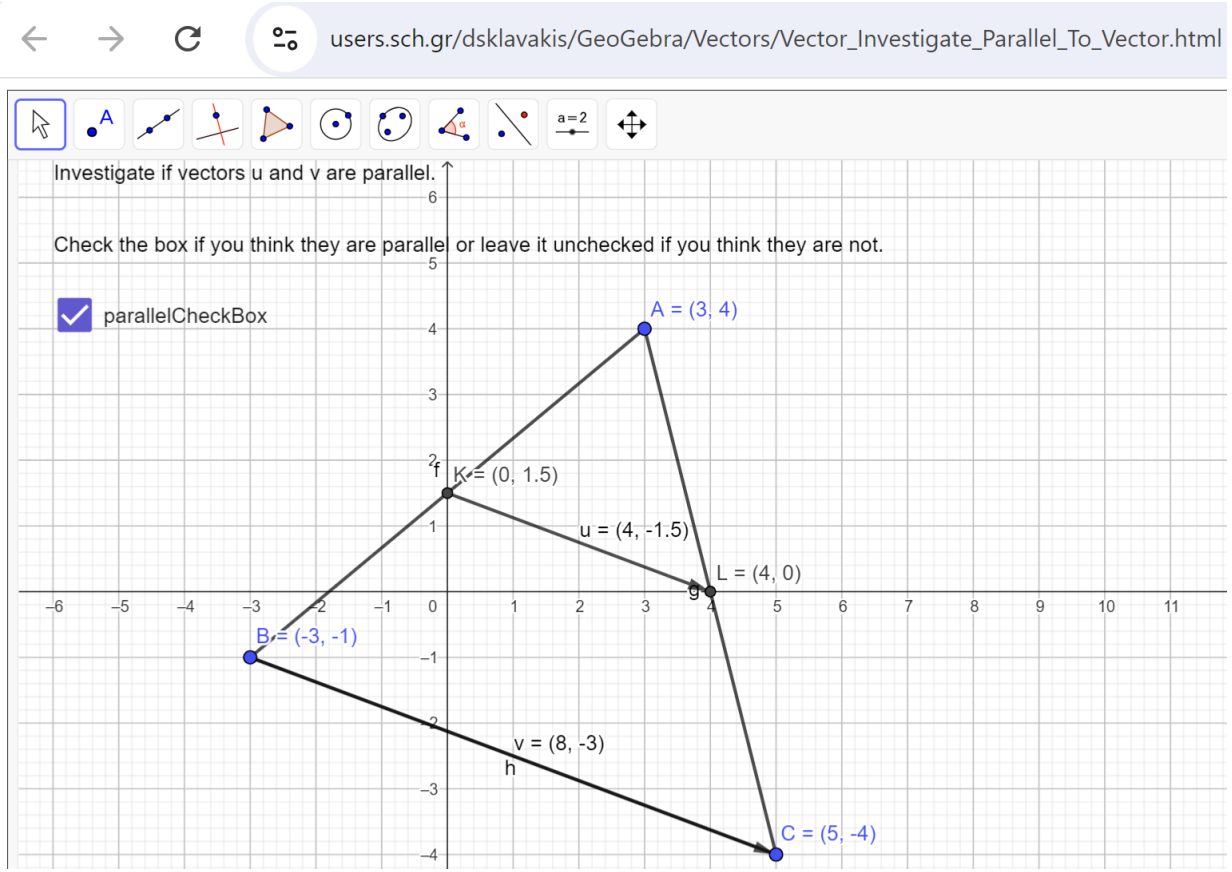
of the sides AB *and* AC correspondingly

2. Show that $\overrightarrow{DE}//\overrightarrow{BC}$

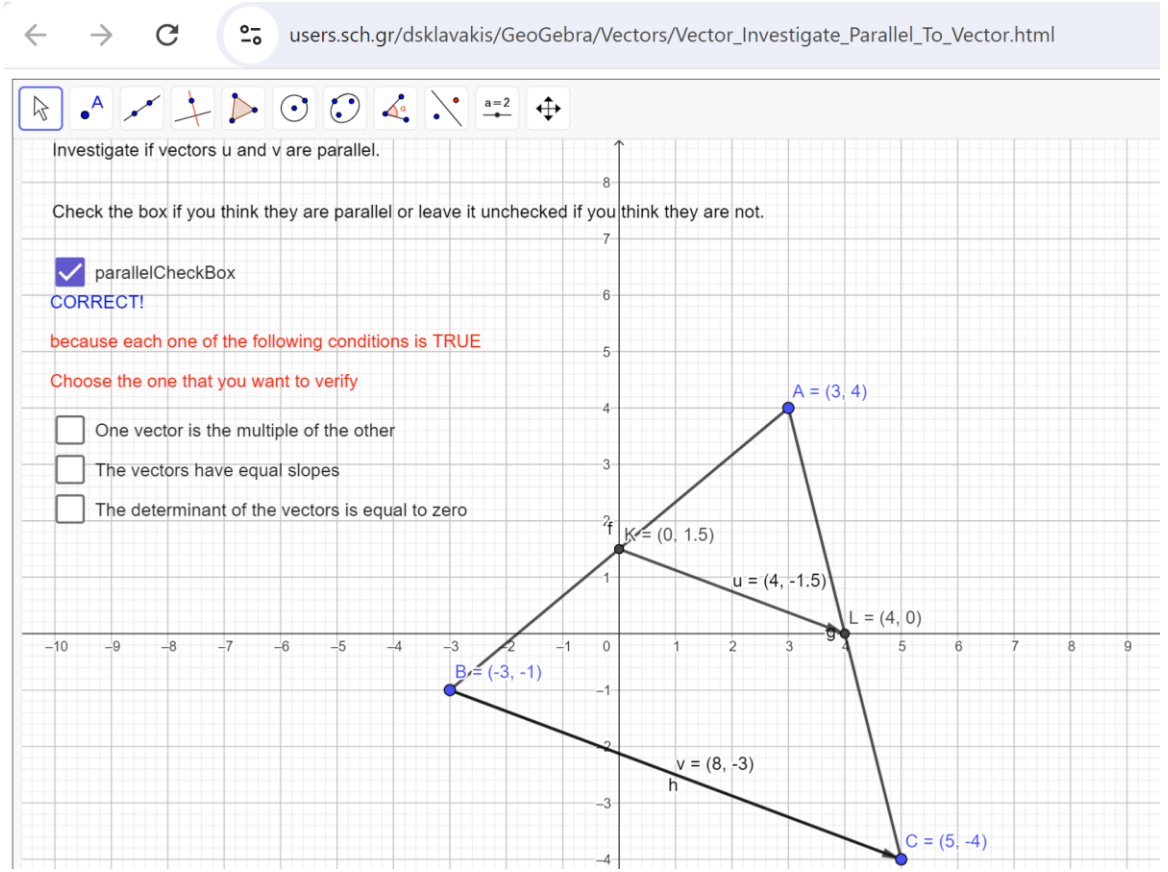


The GeoGebra Intelligent m-Tutors

The Tutoring Model : Deep Model-Tracing (1/3)



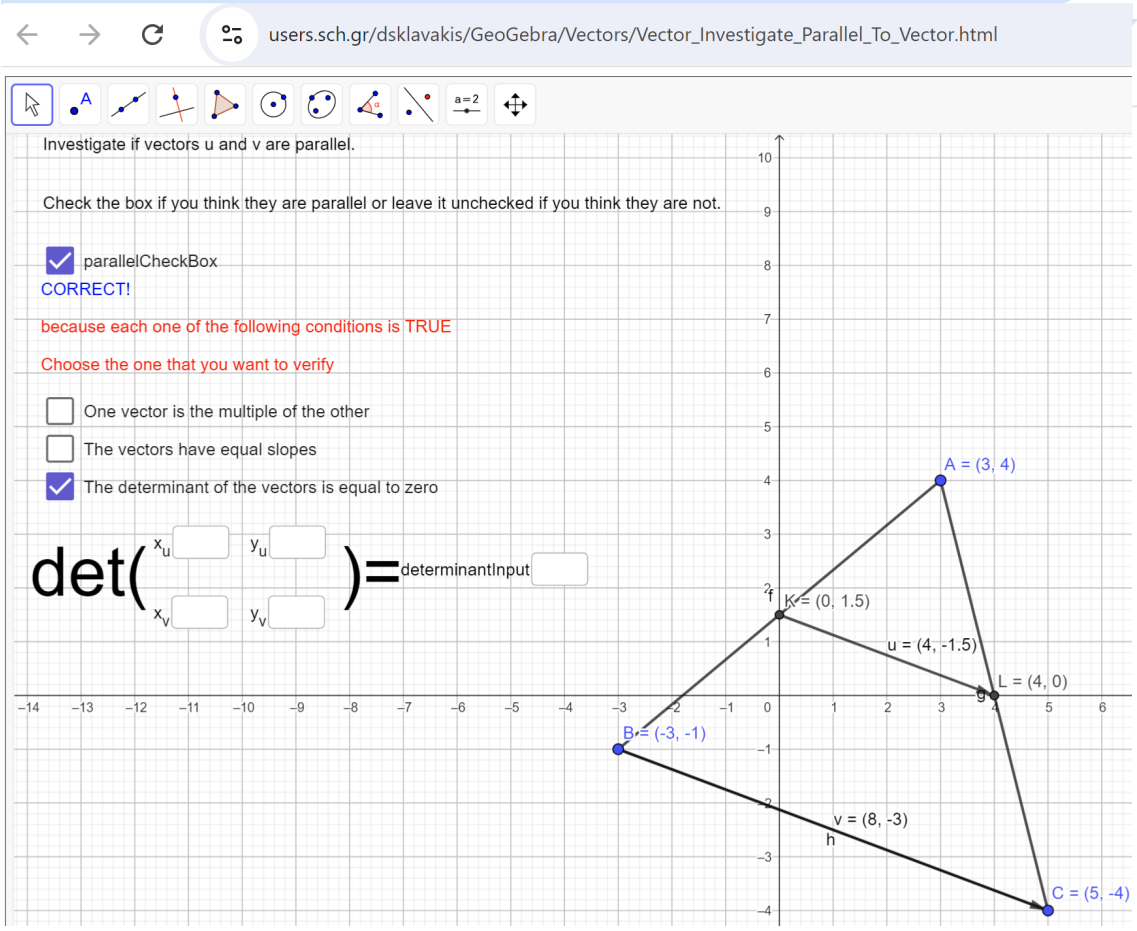
1. The student checks the *parallelCheckBox to indicate that* $\vec{u}\backslash\backslash\vec{v}$.



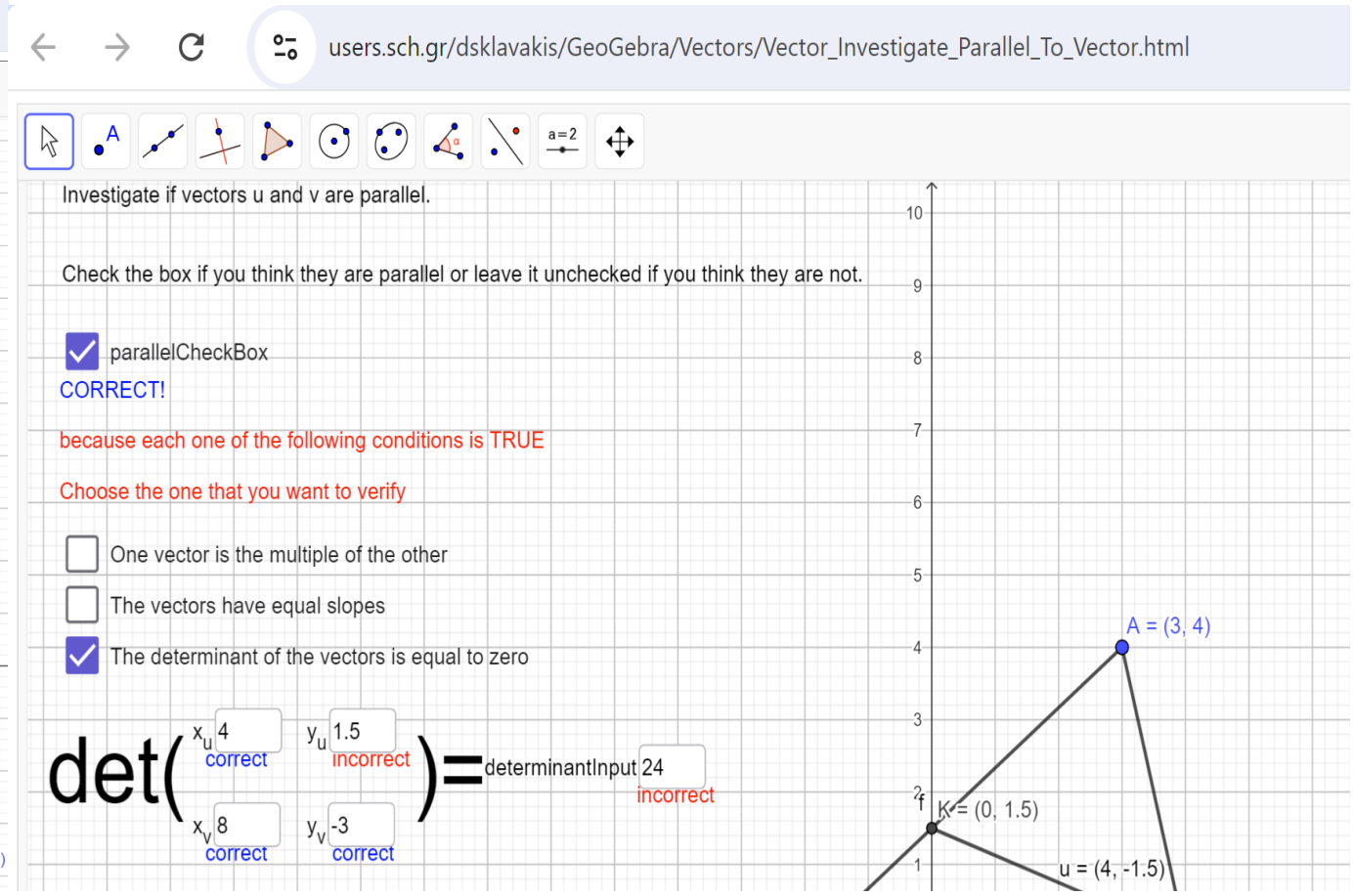
2. The m-Tutor gives positive feedback and asks from the student which method was used

The GeoGebra Intelligent m-Tutors

The Tutoring Model : Deep Model-Tracing (2/3)



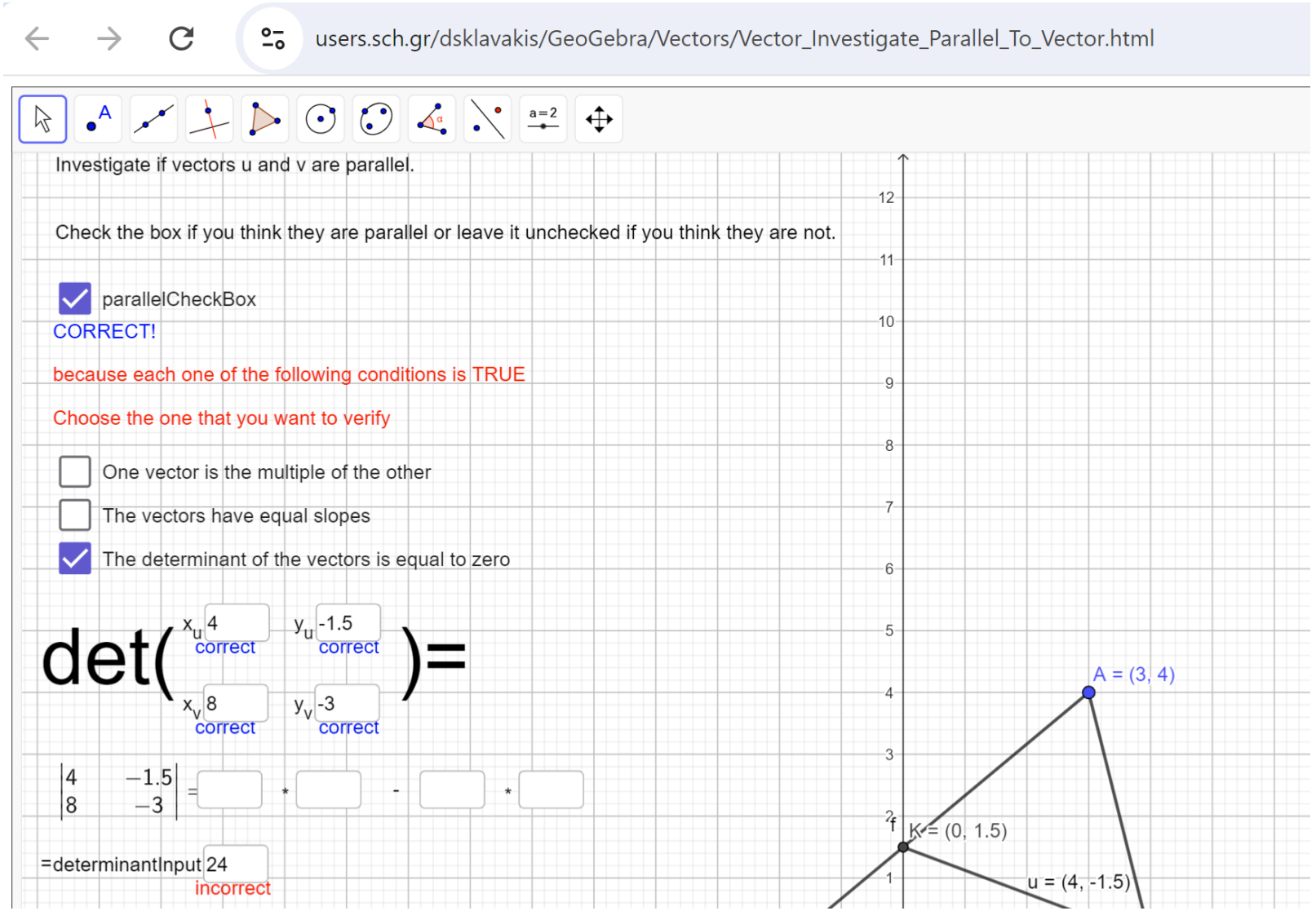
3. The students selects the *zero-determinant* method, and the m-Tutor asks to fill in the coordinates of the vectors and the calculated value of the determinant.



4. The student makes two mistakes, indicated by the m-Tutor by the red colour.

The GeoGebra Intelligent m-Tutors

The Tutoring Model : Deep Model-Tracing (3/3)



5. The student enters correctly the vectors' coordinates but the calculation of the determinant is still wrong.

6. The m-Tutor backtracks even deeper and asks from the student to show how the determinant was actually calculated.

The GeoGebra Intelligent m-Tutors

The Student Model: Competence-Based Records

CompetenceID	Date	Given	Answer	Correct
3 (Segment mid-point)	28/8/2024	A(3,4), B(-3,-1)	(0,3)	0
3	28/8/2024	A(3,4), B(-3,-1)	(0, 0.5)	0
3	28/8/2024	A(3,4), B(-3,-1)	(0, 1.5)	1
6 (Parallel vectors)	28/8/2024	u=(4, 1.5), v=(-8,-3)	parallel	1
5 (Vector length)	28/8/2024	K(0,1.5), L(4,0)	18.25	0
5	28/8/2024	K(0,1.5), L(4,0)	4.27	1

Student competence performance records for formative and summative assessment

The GeoGebra Intelligent m-Tutors

The Student Model: Solution Steps Records

Student Action	JavaScript command for "Playback"
Student selects the m-Tutor *Segment Calculate Midpoint From Start End* from the *Tutor* drop-down menu	`document.getElementById("tutor").value="Segment_Calculate_Midpoint_From_Start_End"`
Student selects point A from the *Starting point* drop-down menu	`document.getElementById("startingPointsList").value="A"`
Student enters 3 as the value of the *xCoordinateInput* input box for the x-coordinate of the midpoint of segment AB	`ggbApplet.setValue("xCoordinateInputVar",3)`
Student clicks the *Check My Answer* button	`checkStudentAnswer()`

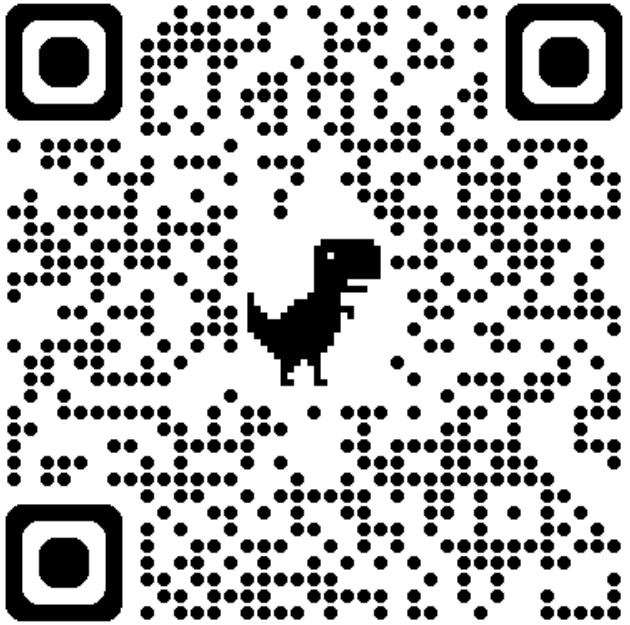
Student actions and JavaScript commands for solution playback

The MATHESIS Learning Management System

Overview

- m-Tutors can be used independently and as stand-alone web-pages,
- MATHESIS integrated Learning Management System (LMS)

https://users.sch.gr/dsklavakis/GeoGebra/MATHESIS.htm



- Student/Teacher management: Sign Up / Sign In
- Class management: Create, Insert Student, Delete Student
- Booklet management: Create, Insert, Annotate, Delete exercises
- Assignment of exercises to students
- Playback and Assessment of student assignments both for teachers and students

The GeoGebra Intelligent m-Tutors

Further Work

1. Extend the Domain Expertise Model
   - Calculation of the point of intersection of two lines
   - Calculation of a line's equation parallel/vertical to another line
   - Calculation of the acute angle of two lines
   - Calculation of the distance of a point from a line;
   - Calculation of the projection of a point to a line
   - Calculation of the distance of two parallel lines.

2. Authoring Tools for Solution Plans of the top-level competences
   - The student selects the top-level competence to perform and the order of performance
   - The Authoring Tools will allow Tutors to construct a Semantic Diagram that will define the correct/acceptable order of the competences to be performed