CONTINUOUS FUNCTIONS

SOLVED EXCERCISES

1) Is f continuous at x_0, where: α) x_0=-4, β) x_0=0, γ) x_0=4, δ) x_0=5 (see fig.1)? Can we examine the continuity of f at x_0=2?

Solution:

α) Since $\lim_{x\to -4^+} f(x)=0=f(-4)$ f is continuous at -4.

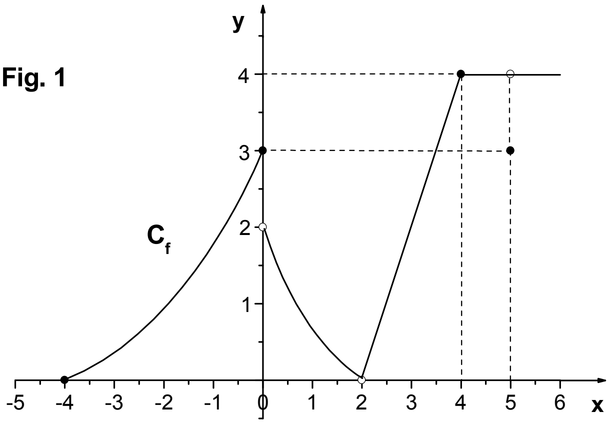
β) We have $\lim_{x\to 0^-} f(x)=3$ but $\lim_{x\to 0^+} f(x)=2$ thus there is not $\lim_{x\to 0} f(x)$, i.e f is not continuous at 0.

γ) Since $\lim_{x\to 4^+} f(x)=\lim_{x\to 4^-} f(x)=f(4)$ f is continuous at 4.

δ) f is not continuous at 5, since $\lim_{x\to 5} f(x)=4$ but f(5)=3.

We examine the continuity at a point which belongs to the domain of a function. The number 2 does not belong to the domain of f.

Fig. 1



2) Examine the continuity of f at x_0:

α) $f(x)=\begin{cases} |x|, & x<0 \\ x, & 0\le x<2 \\ x^2, & x\ge 2 \end{cases}$ $x_0=0$, $x_0=2$ **β)** $f(x)=\begin{cases} \frac{1}{x^2}, & x\neq 0 \\ 0, & x=0 \end{cases}$ $x_0=0$ **γ)** $f(x)=|x|+5$, $x_0=0$.

Solution:

α) Since $\lim_{x\to 0^-} f(x)=\lim_{x\to 0^-}|x|=0$, $\lim_{x\to 0^+} f(x)=\lim_{x\to 0^+} x=0$, f(0)=0 f is continuous at x_0=0.

We have that $\lim_{x\to 2^-} f(x)=\lim_{x\to 2^-} x=2$, $\lim_{x\to 2^+} f(x)=\lim_{x\to 2^+} x^2=4$, and consequently there is not the limit of f at x_0=2, thus f is not continuous at 2.

β) $\lim_{x\to 0}\frac{1}{x^2}=+\infty\notin \mathbf{R}$ but f(0)=0, thus f is not continuous at 0.

γ) Since $\lim_{x\to 0} f(x)=\lim_{x\to 0}\left(|x|+5\right)=0+5=5=f(0)$, f is continuous at 0.